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COSMIC ACCELERATION: FROM THE COSMOLOGICAL CONSTANT TO DARK ENERGY AND MODIFIED GRAVITY THEORIES

Astrophysical probes of fundamental physics
ferrara, 7-11 september 2015

- Basics of structure formation in Dark Energy models
    - Homogeneous Dark Energy
    - Clustering Dark Energy
    - Coupled Dark Energy
    - *f(R)* gravity
- N-body simulations
    - Basics of numerical simulations of structure formation
    - Modified algorithms for Dark Energy models
    - Modified algorithms for Modified Gravity models
- Non-linear structure formation in Dark Energy models
    - DE parameterisations and Early Dark Energy
    - Coupled Quintessence
    - Dark Scattering
- Non-linear structure formation in Modified Gravity models
    - *f(R)* simulations
    - The degeneracy with massive neutrinos

BASICS OF STRUCTURE FORMATION IN DARK ENERGY MODELS

Classification of Dark Energy models

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	time evolution	spatial fluctuations	interactions
Λ	✘	✘	✘
Dynamical DE (DE parameterisations, Quintessence, k-essence)	✔ a dynamical (scalar) degree of freedom	✘ no clustering at sub-horizon scales	✘ minimally-coupled to matter fields

Homogeneous Dark Energy models (I)

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Scalar field models

A scalar degree of freedom $\phi(t)$ evolving in a self-interaction potential

$$\rho_{\rm DE} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad c_\phi^2 \approx 1$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Wetterich 1988; Ratra & Peebles 1988
Ferreira & Joyce 1998; Brax & Martin 1999

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Dark Energy Parameterisations

A parametrization of the time evolution of DE

Time-dependent equation of state (Chevalier-Polarski-Linder):

$$w_{\rm DE}(a) = w_0 + w_a(1-a)$$

Early Dark Energy (Wetterich 2004):

$$w_{\rm DE}(a) = \frac{w_0}{1 + b\ln(1/a)} \qquad b = -\frac{3w_0}{\ln\frac{1-\Omega_{\rm EDE}}{\Omega_{\rm EDE}} + \ln\frac{1-\Omega_M}{\Omega_M}}$$

Homogeneous Dark Energy models (II)

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A homogeneous and minimally-coupled DE scalar field will affect structure formation only through the background expansion history of the Universe:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_M a^{-3} + (1-\Omega_M)\exp\left(-3\int_1^a \frac{1+w(a')}{a'}da'\right) \quad (94)$$

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Therefore, gravitational interactions are not directly affected:

$$c_\phi^2 \approx 1 \Rightarrow \delta\rho_\phi \approx 0$$ ⟵ no DE perturbations at sub-horizon scales

$$\nabla^2\Phi_g = -4\pi G\delta\rho_M$$ ⟵ standard Poisson equation

$$\vec{a} = -\vec{\nabla}\Phi_g$$ ⟵ no additional forces beyond standard gravity

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Clustering DE ("cold" DE models, Unified DE models)	✔ a dynamical (scalar) degree of freedom	✔ small sound speed, clustering at sub-H	✘ minimally coupled to matter

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Nonetheless, massive particles still follow geodesics since there is no modification of the gravitational interaction:

$$\vec{a} = -\vec{\nabla}\Phi_g$$

← no additional forces beyond standard gravity

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Interacting DE (Coupled and Extended Quintessence, Modified Gravity)	✔ a dynamical (scalar) degree of freedom	✔ fluctuations sourced by the interaction	✔ non-minimally coupled to matter

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We have seen that an interaction between a matter field and a scalar field can be described by a source term in the respective continuity equations:

$$\nabla_\mu T^{\mu(\phi)}_\nu = -QT^{(\mathrm{DM})}\nabla_\nu\phi \qquad \nabla_\mu T^{\mu(\mathrm{DM})}_\nu = +QT^{(\mathrm{DM})}\nabla_\nu\phi$$

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By perturbing these equations at linear order one obtains a dynamical equation for the field perturbations:

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that in the so-called quasi-static limit ($\partial^2\delta\phi/\partial t^2 \ll \nabla^2\delta\phi$) gives:

$$\nabla^2\delta\phi = -\frac{dV}{d\phi}(\delta\phi) - AQ\delta_{\mathrm{DM}} \tag{97}$$

and assuming a flat potential ($dV/d\phi \ll \delta_{\mathrm{DM}}$):

$$\nabla^2\delta\phi \approx -AQ\delta_{\mathrm{DM}} \Rightarrow \boxed{\delta\phi \approx -AQ\Phi} \tag{98}$$

Interacting Dark Energy models (II)

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By combining this relation with the linear perturbations equations of the matter field, one obtains a modified version of the gravitational instability equation (in the cosmic time t):

$$Q = 0 \qquad \delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega\mathcal{H}^2\delta = 0 \longrightarrow \ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega\delta = 0 \tag{99}$$

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As we know that scalar fifth-forces are tightly constrained from solar system tests of gravity, it is necessary to allow for a non-universal coupling so that cold dark matter is coupled and baryons are uncoupled ($Q_c \neq 0, \; Q_b = 0$). In this case one gets two separate gravitational instability equations:

$$\ddot{\delta}_c + \left(2H - 2Q\dot{\phi}\right)\dot{\delta}_c - \frac{3}{2}H^2\left[\left(1 + 2Q^2\right)\Omega_c\delta_c + \Omega_b\delta_b\right] = 0 \tag{101}$$

$$\ddot{\delta}_b + 2H\dot{\delta}_b - \frac{3}{2}H^2\left[\Omega_c\delta_c + \Omega_b\delta_b\right] = 0 \tag{102}$$

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This implies an effective violation of the Weak Equivalence Principle:

$$\vec{a}_{\rm CDM} = -\vec{\nabla}\Phi(1+2Q^2)+2Q\dot{\phi}\vec{v}_{\rm CDM} \qquad \vec{a}_{\rm b} = -\vec{\nabla}\Phi \tag{103}$$

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The term $2Q\dot{\phi}\vec{v}_{\rm CDM}$ is called "friction term" and arises from momentum conservation:

$$\frac{d\vec{p}}{dt} = \frac{d(m(\phi)\vec{v})}{dt} = m(\phi)\vec{a}+\frac{dm}{d\phi}\dot{\phi}\vec{v} \tag{104}$$

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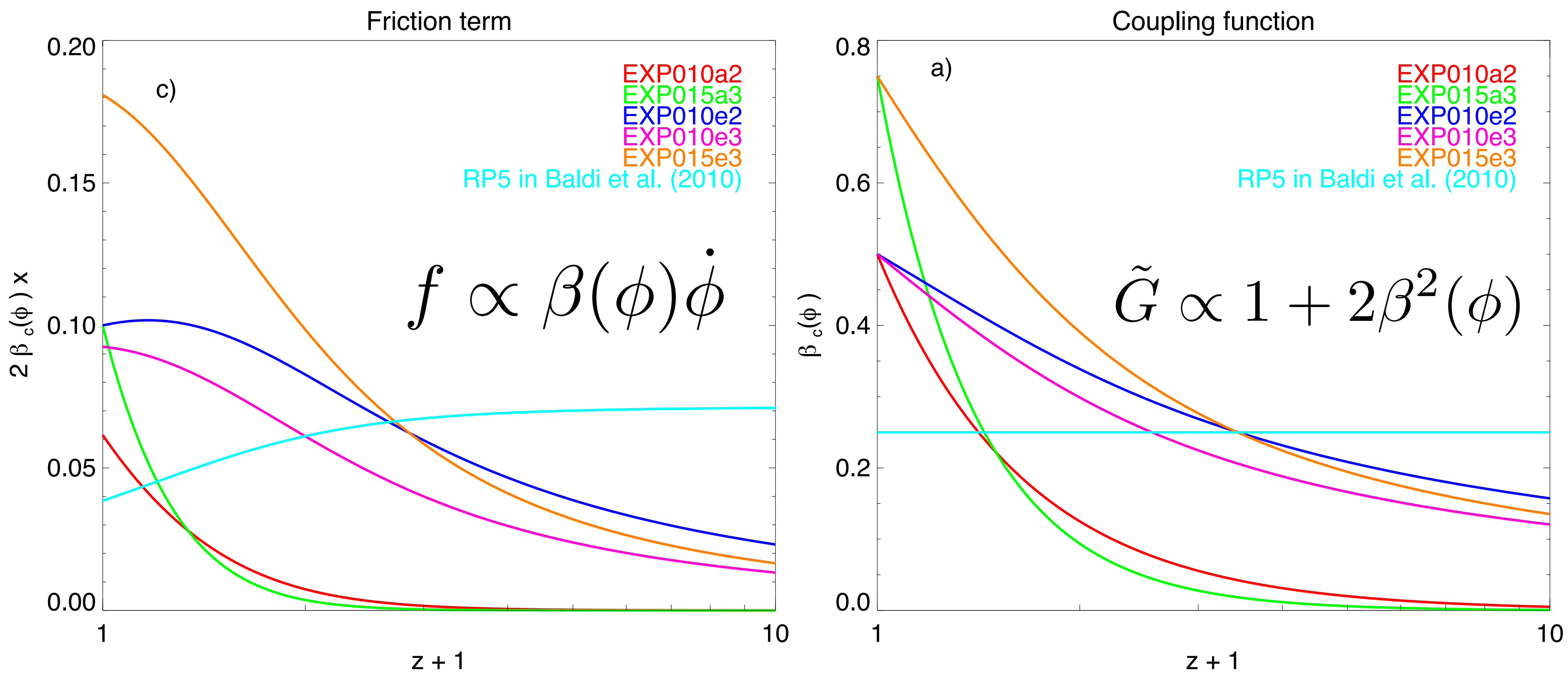
For constant and field-dependent couplings these two terms look like



Baldi 2012

Modified Gravity models: *f(R)* (I)

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A similar situation (with some significant differences) applies to modified gravity models. In the case of *f(R)* gravity, we have seen that the trace of the modified Einstein equations gives:

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By perturbing this equation at linear order (posing $f_R = \bar{f}_R + \delta f_R$):

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta f_R = -M^2(f_R)\delta f_R + \frac{8\pi G}{3\bar{f}_R}\delta\rho_{\mathrm{M}} \qquad (105)$$

That in the quasi-static approximation becomes:

$$\boxed{\nabla^2\delta f_R = M^2(f_R)\delta f_R - \frac{8\pi G}{3\bar{f}_R}\delta\rho_{\mathrm{M}}} \qquad (106)$$

that is the same equation we saw for interacting dark energy.
However, differently from the case of interacting Dark Energy, in f(R) one CANNOT assume

$$M^2(f_R)\delta f_R \ll \delta\rho_{\mathrm{M}}$$ **NOT TRUE**

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By combining the dynamic equation for δf_R with the evolution equations of linear matter density perturbations, one gets the gravitational instability equation for *f(R)* gravity in the small scale limit, that takes the form:

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from which we can see that gravity is enhanced by a factor 4/3 which corresponds to a coupling Q for interacting DE:

$$1 + 2Q^2 = 4/3 \Rightarrow \boxed{Q = \frac{1}{\sqrt{6}}} \tag{108}$$

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However, the variability of the $f(R)$ function is hidden in the definition of the $\tilde{\Omega}_{\rm M}$ parameter:

$$\tilde{\Omega}_{\rm M} \equiv \frac{8\pi G\rho_{\rm M}}{3\bar{f}_R H^2} \tag{109}$$

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Such a property can be realised in several ways. The screening mechanism that occurs in f(R) gravity models is called Chameleon

Chameleon screening (Khoury & Weltman 2004):

the scalar mass $m_{f_R} \propto dM^2/df_R$ becomes large in high-density regions of the Universe, so that the fifth-force does not propagate and standard GR is restored

$$\vec{a}(r) = -\vec{\nabla}\Phi - Q(\phi)e^{-m_\phi r}\vec{\nabla}\delta\phi \quad (110)$$

WHY DO WE NEED SIMULATIONS?

The non-linear regime of structure formation

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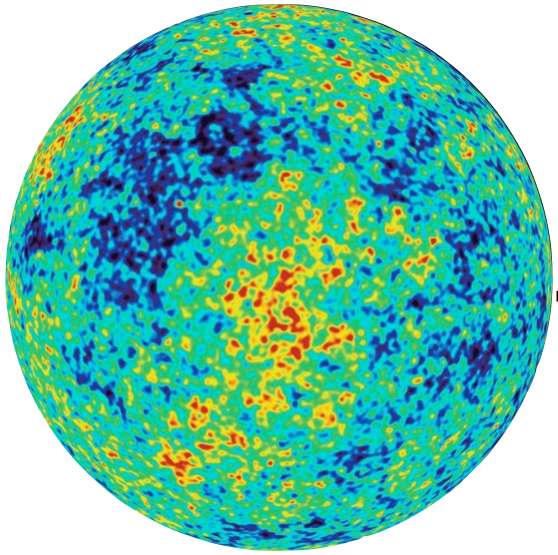
Primordial density field

$z_{\rm CMB} \approx 10^3, a_{\rm CMB} \approx 10^{-3}$



$\Delta T/T \approx \delta\rho_b/\rho_b \approx 10^{-5}$

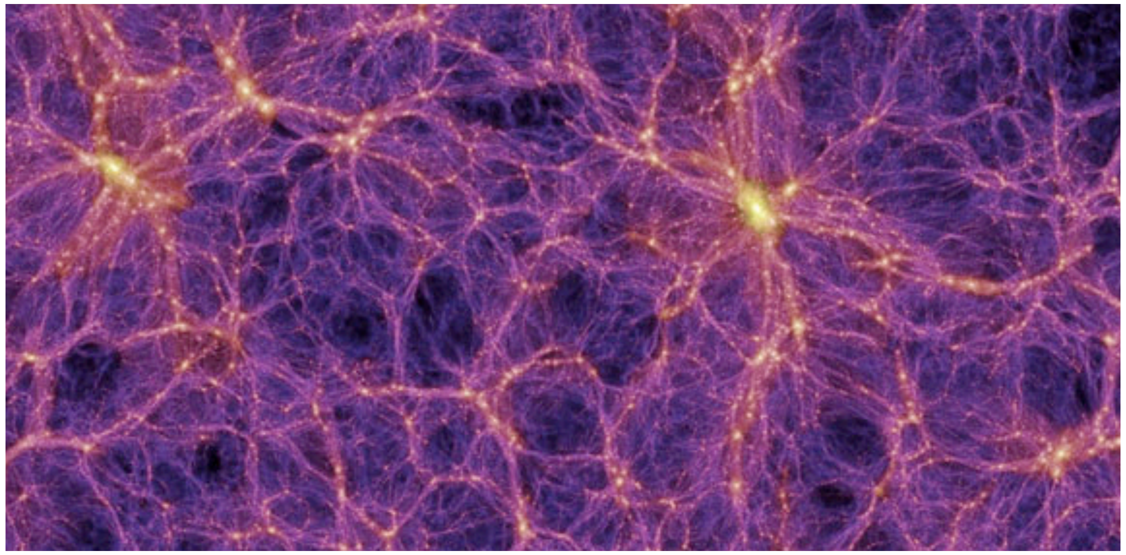
gravitational instability

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}^2\delta = 0$$

$$D_+ \approx 10^3$$

Structures in the present-day Universe

$z_0 = 0, a_0 = 1$



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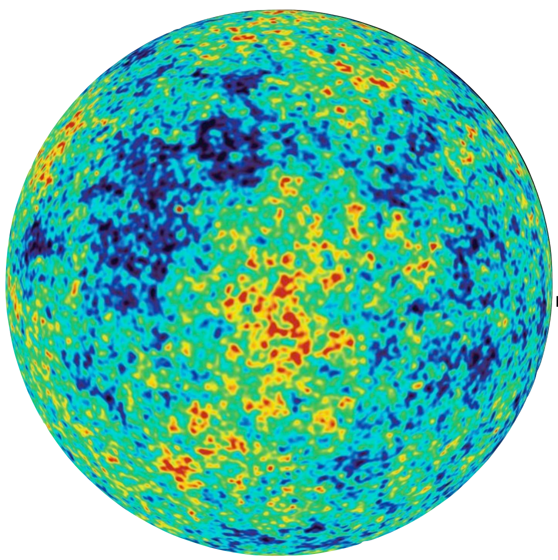

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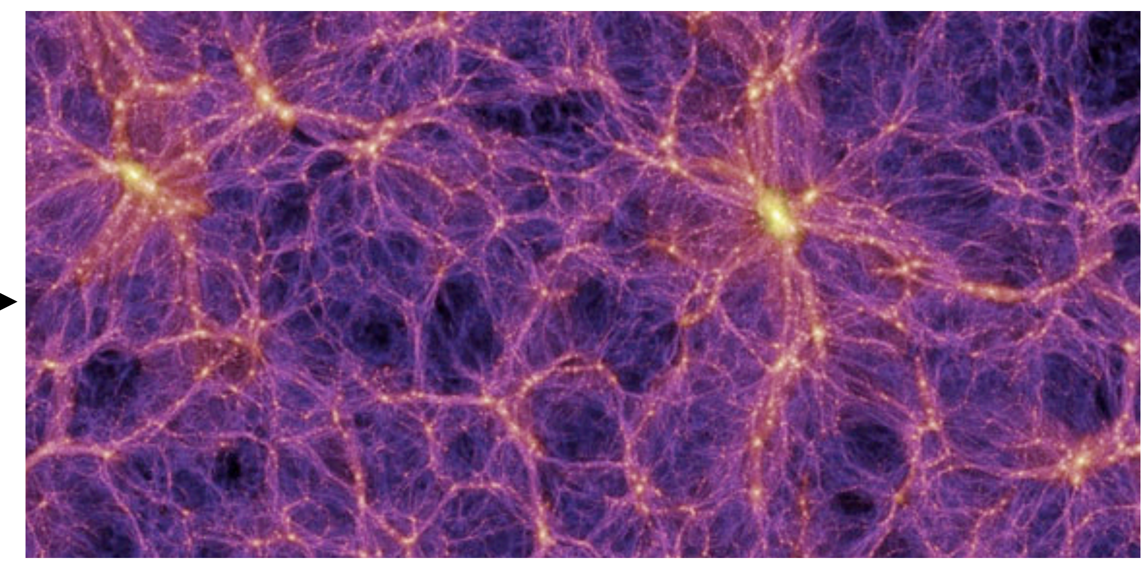

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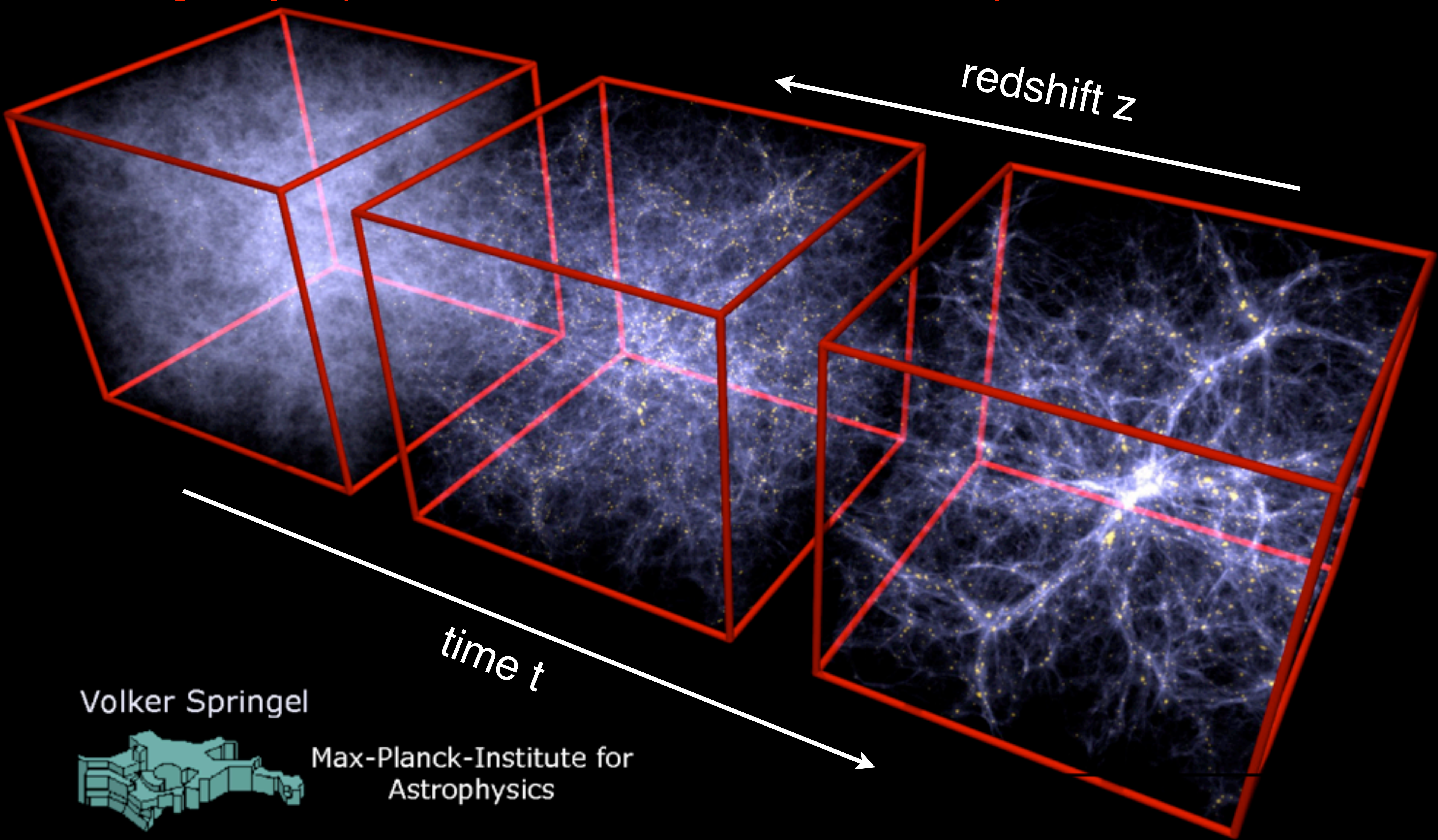
... we know that in the present Universe density perturbations can reach large values ($\delta \approx 1$ on scales of $\sim 8{\rm Mpc}$ and up to $\delta \approx 10^5$ in the center of galaxy clusters).

The assumption of small perturbations does not hold anymore, and linearity no longer applies $\longrightarrow$ need of numerical methods

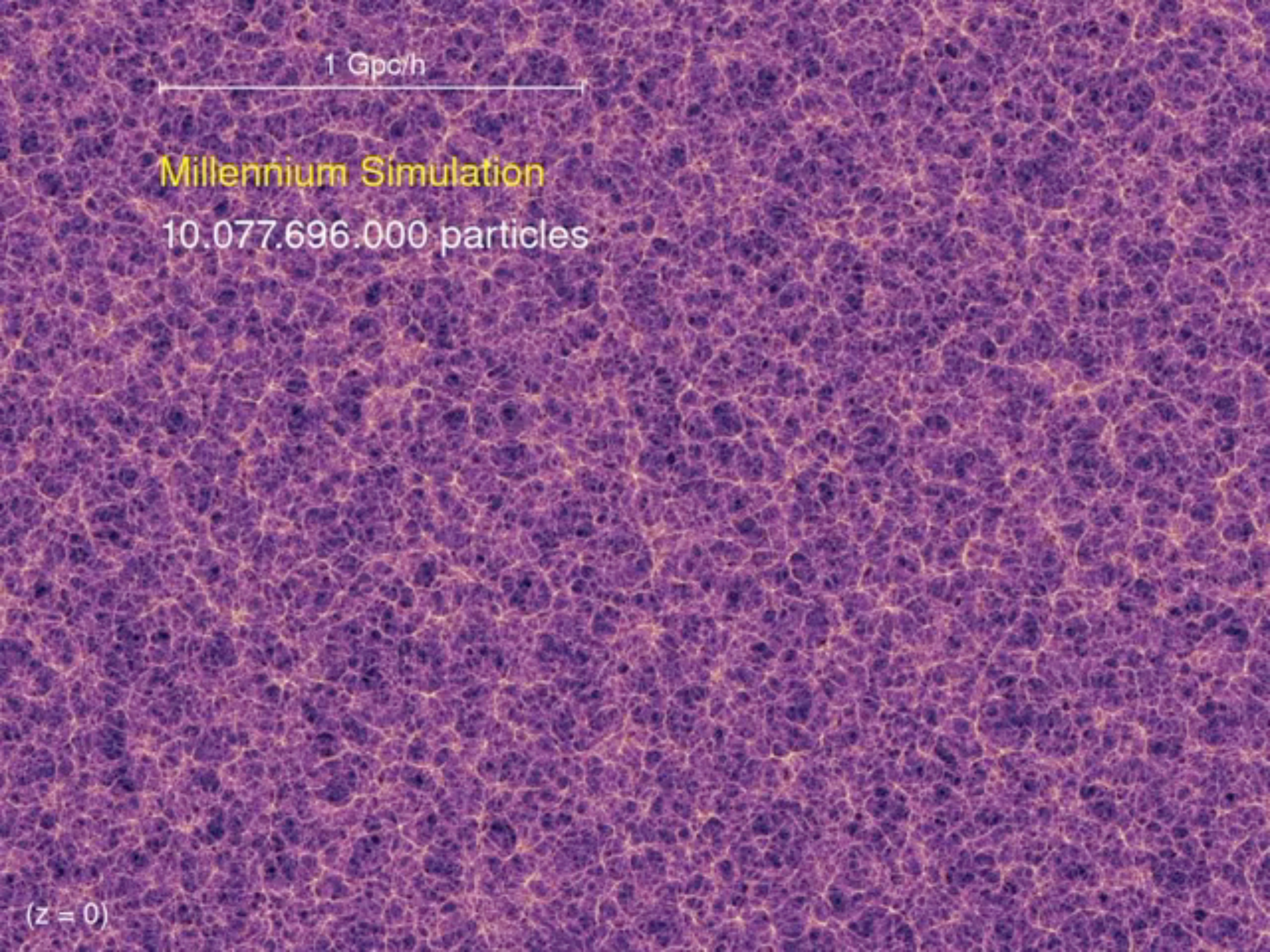
N-BODY SIMULATIONS

Cosmological N-body simulations

Integrate the evolution of density perturbations forward in time (starting from a known initial power spectrum) within a periodic, comoving, and cosmologically representative box filled with tracer particles



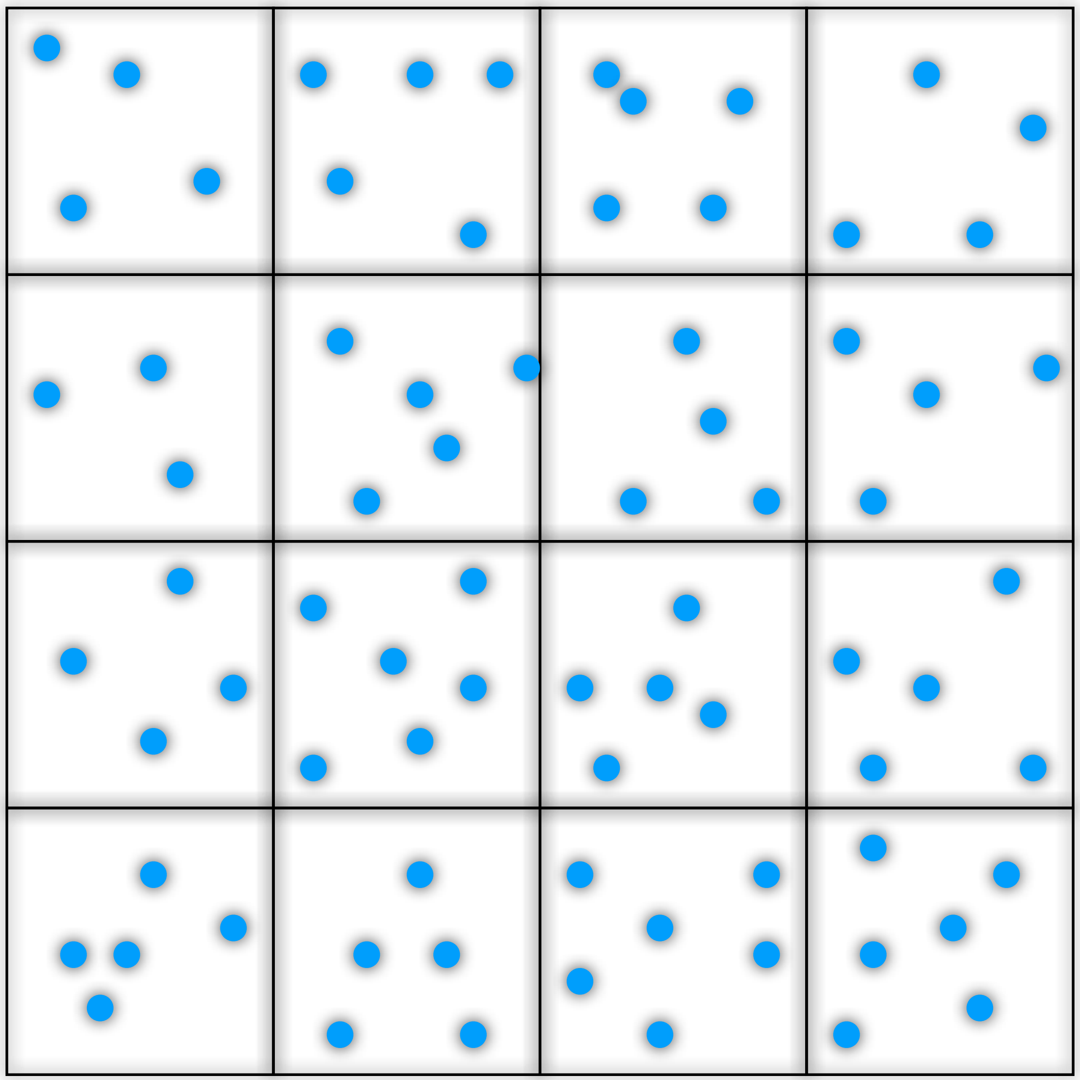

z = 48.4
T = 0.05 Gyr
500 kpc
Millennium Run
Springel et al. 2005


1 Gpc/h
Millennium Simulation
10.077.696.000 particles
(z = 0)

N-body algorithms in pills (I)

The nonlinear regime of structure formation could possibly probe the largest deviation from ΛCDM: need of N-body!

Particle-Mesh



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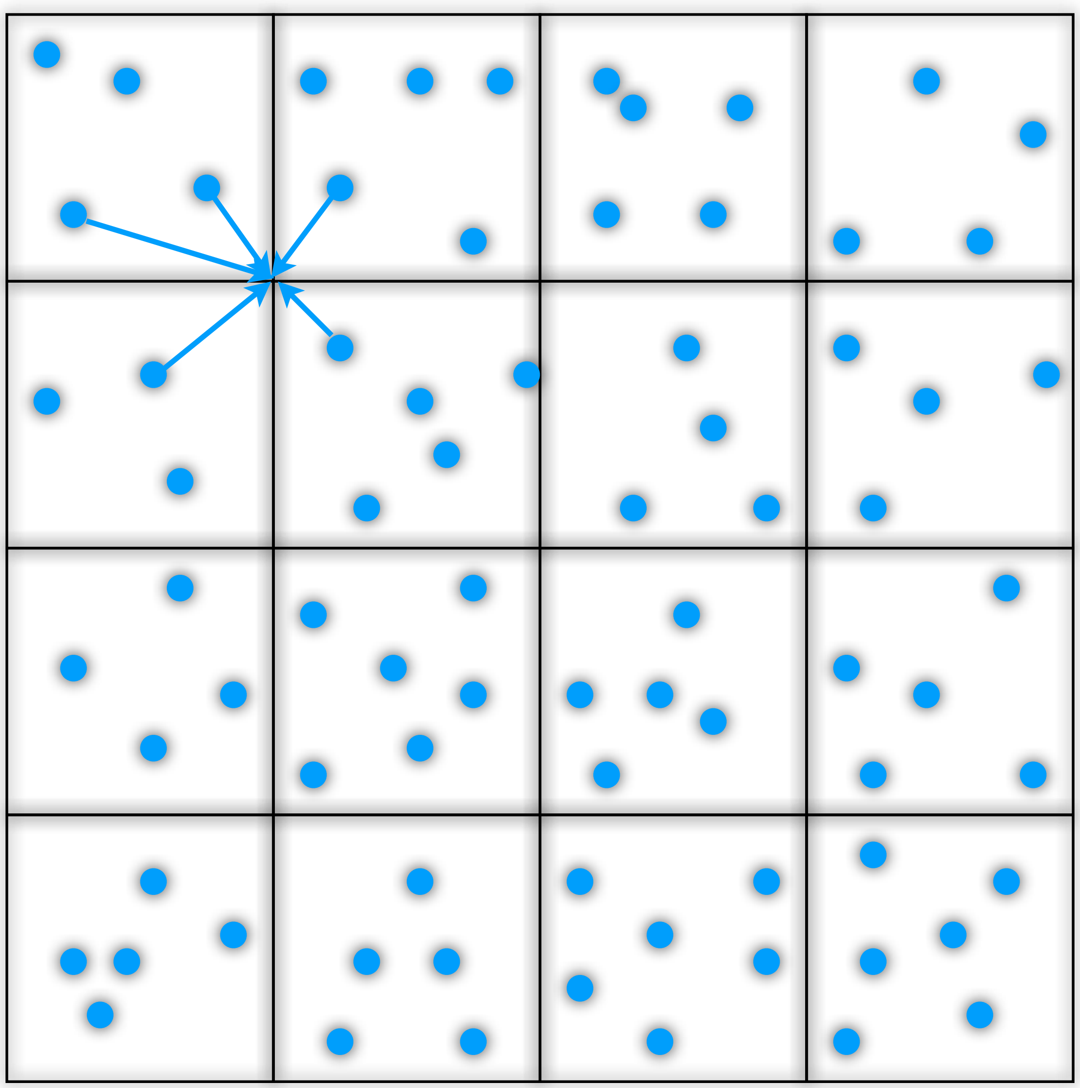

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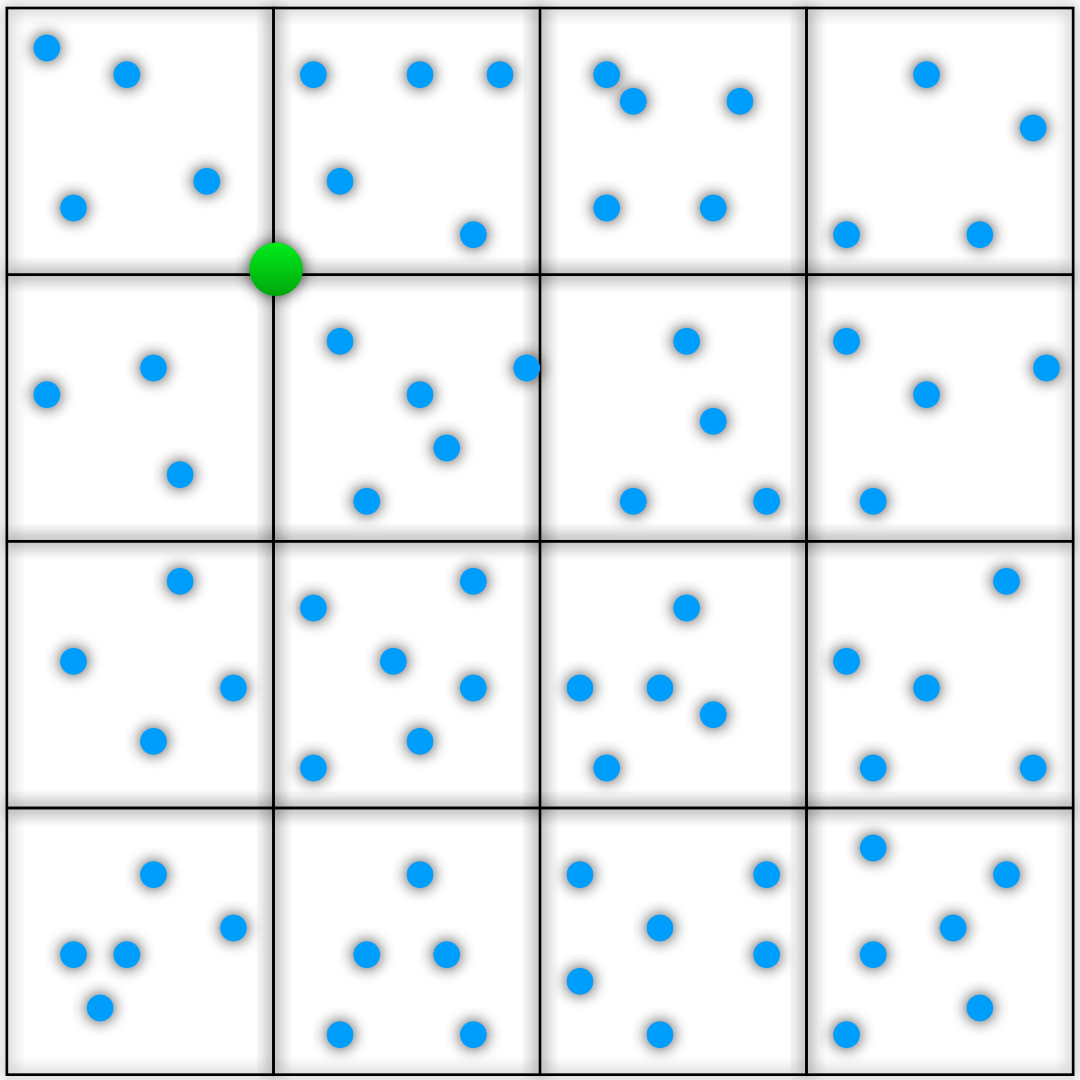

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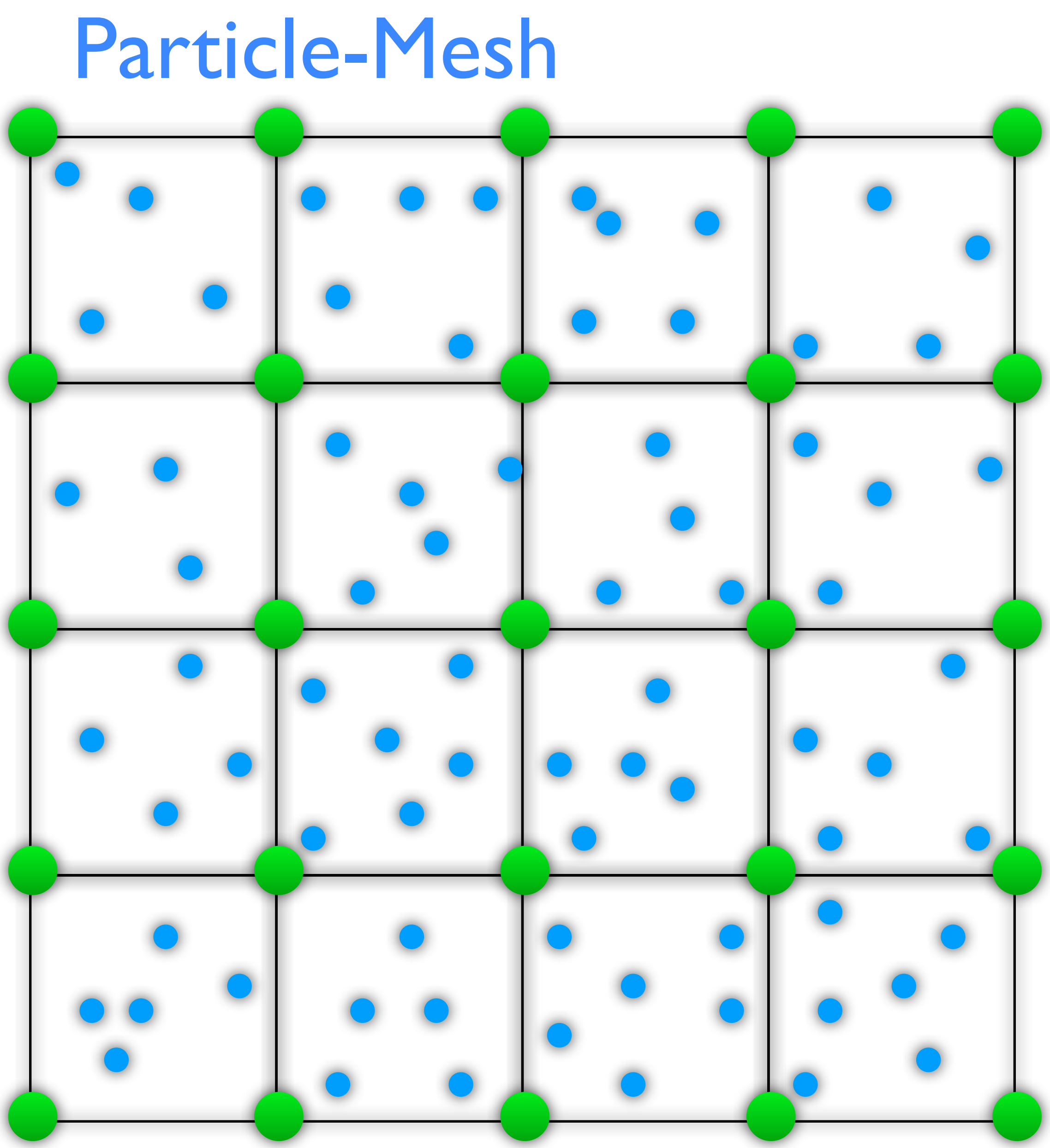

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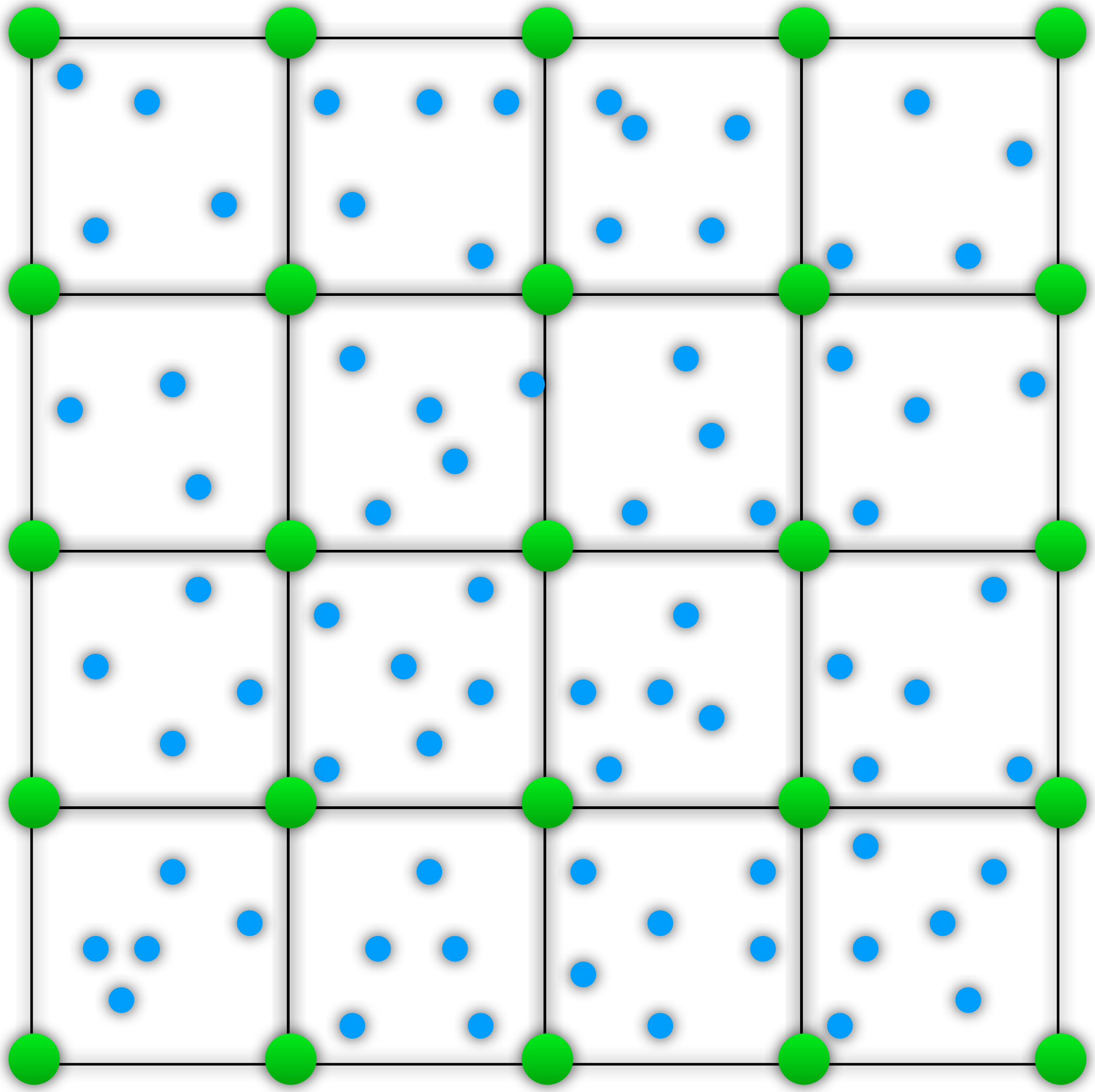

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2) In Fourier space, compute the gravitational potential with the Newtonian Green's function $1/k^2$

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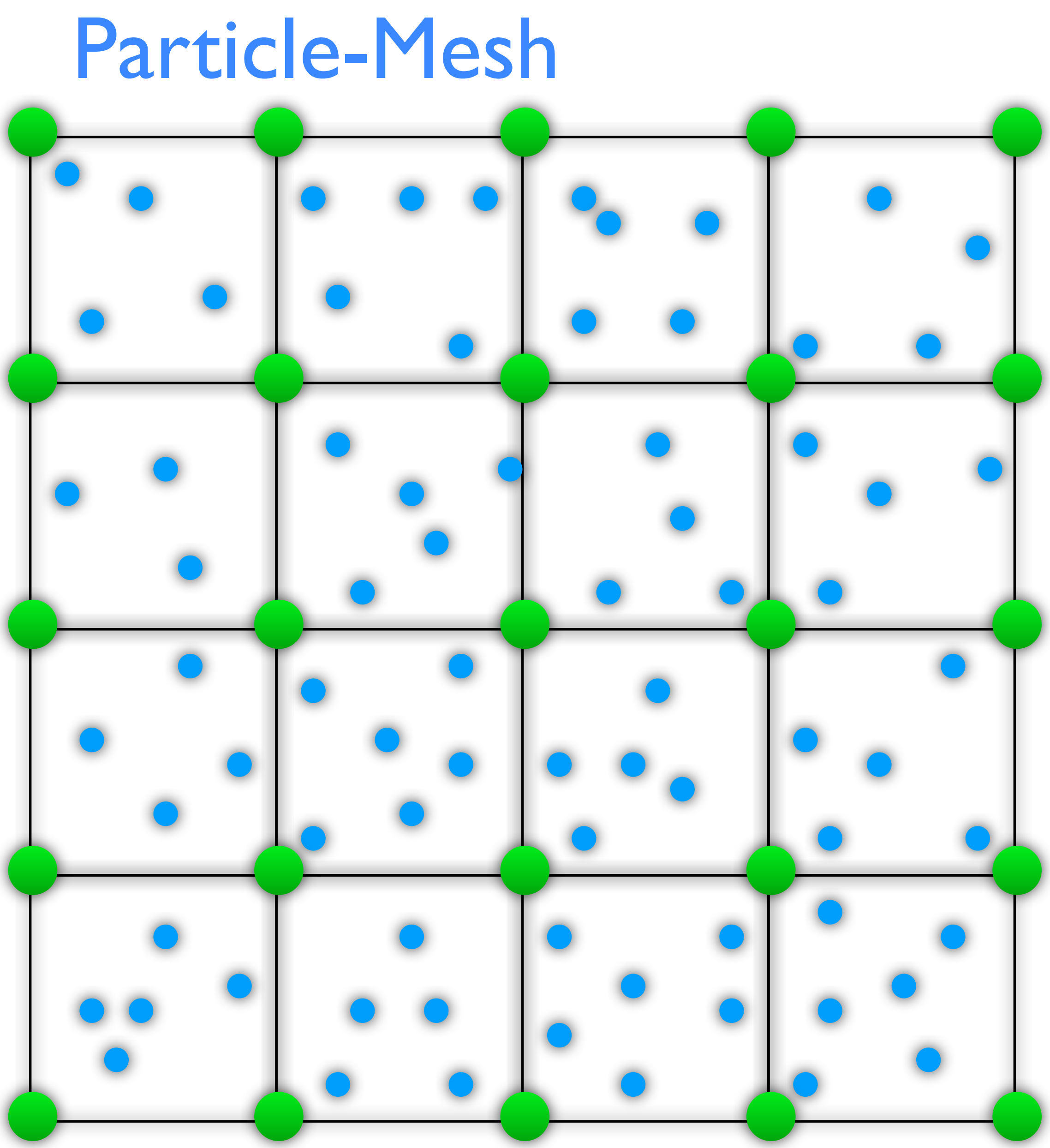

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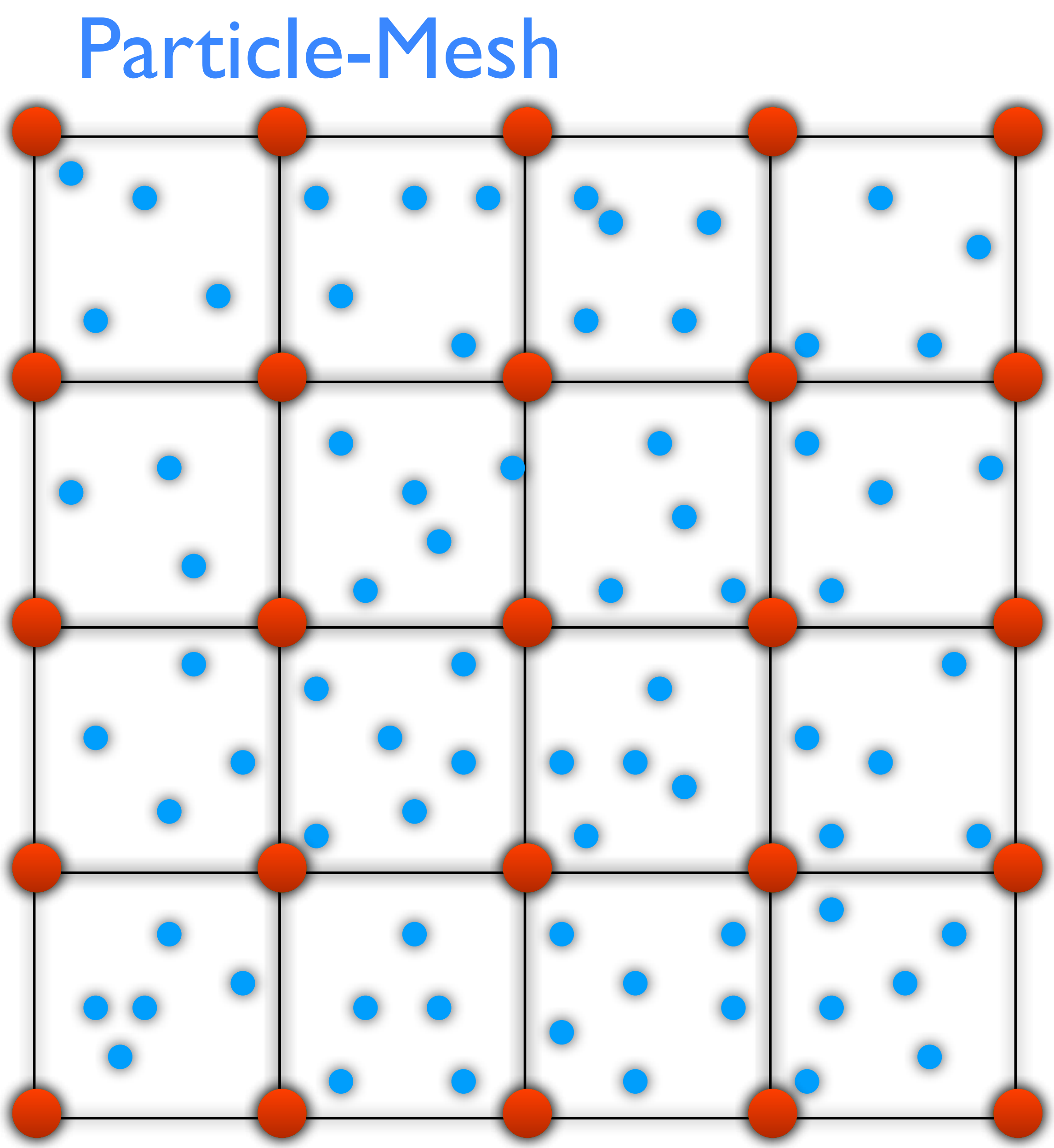

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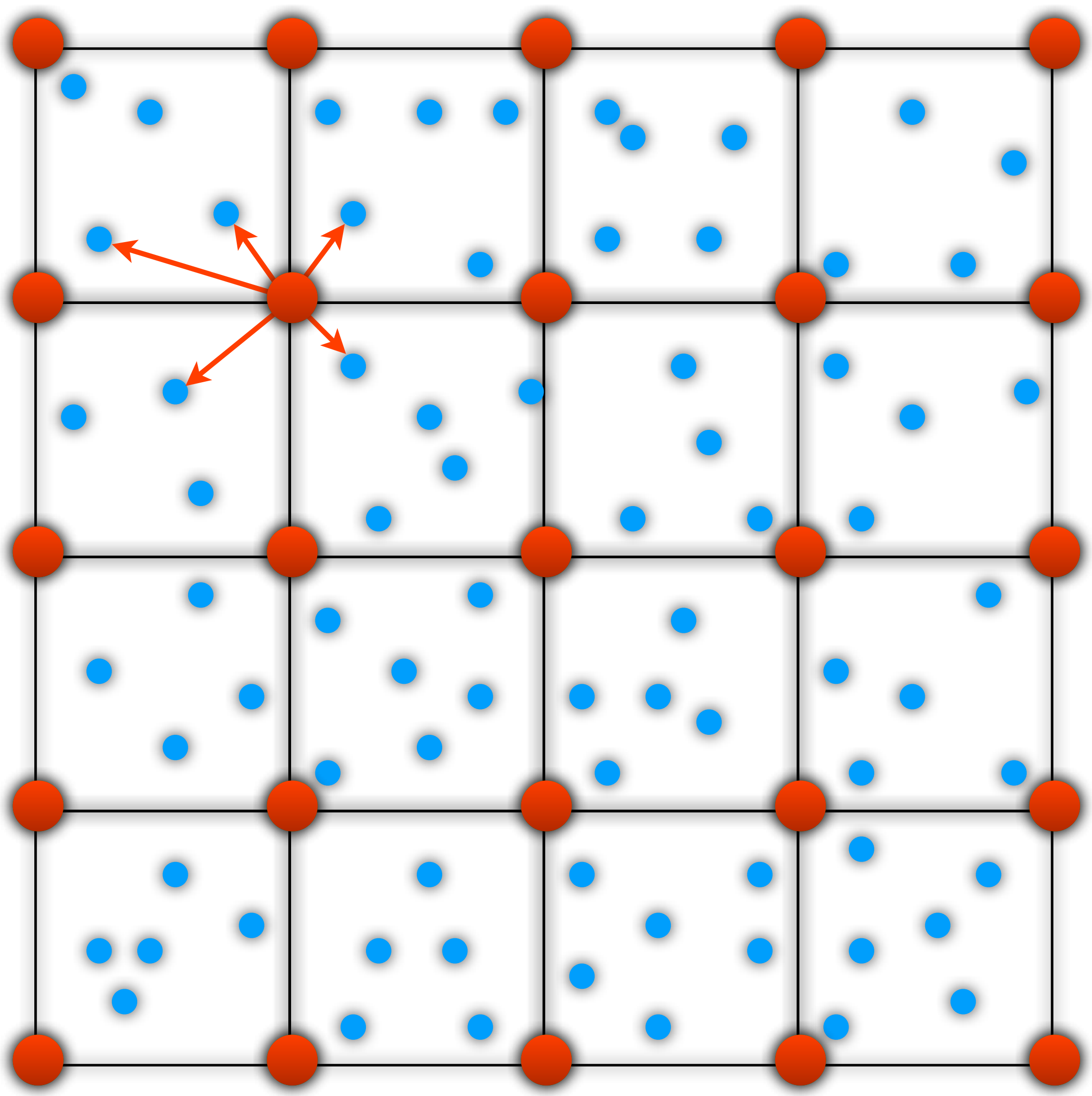

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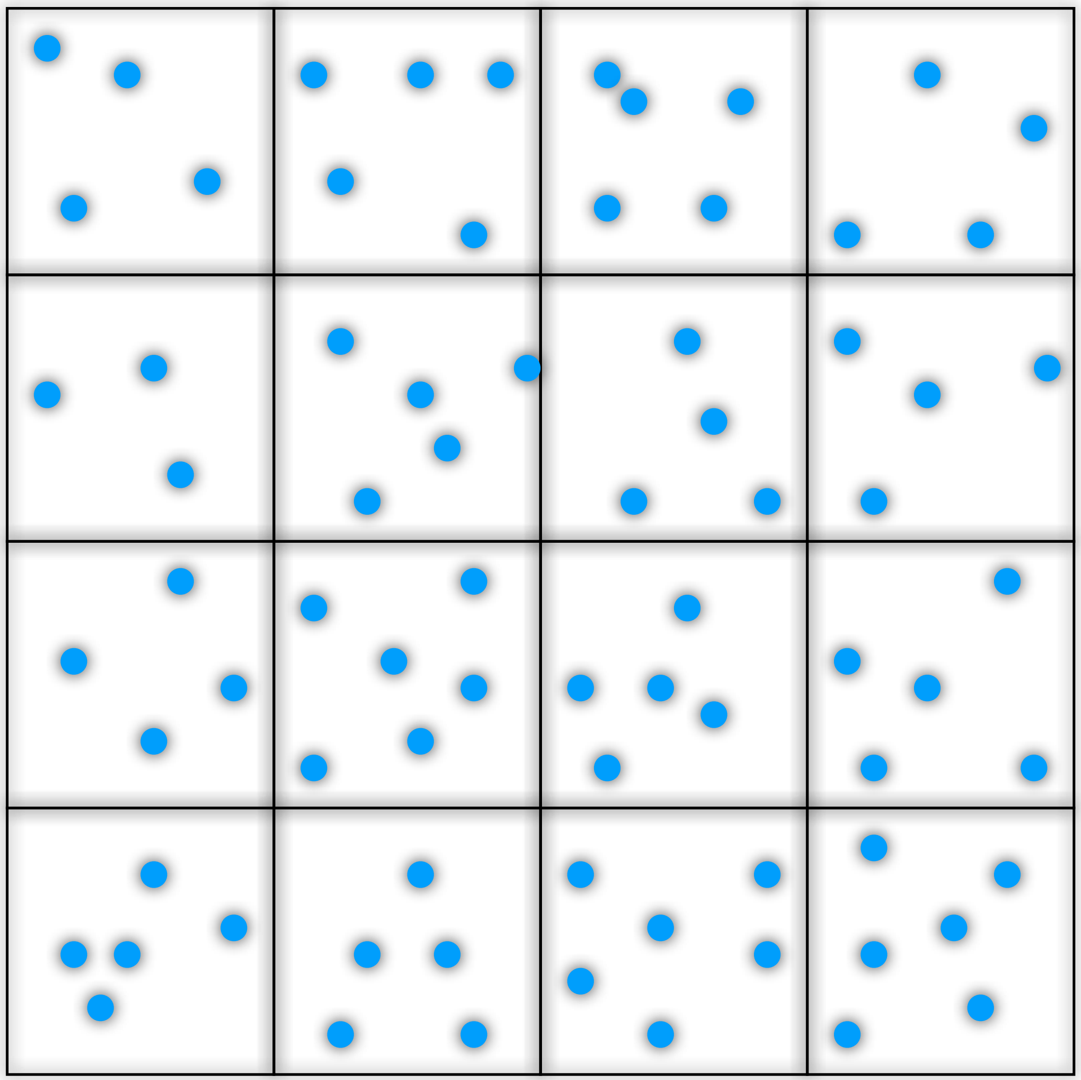
N-body algorithms in pills (II)

Particle-Particle (Tree) ⇒ N^2 problem (N logN problem)



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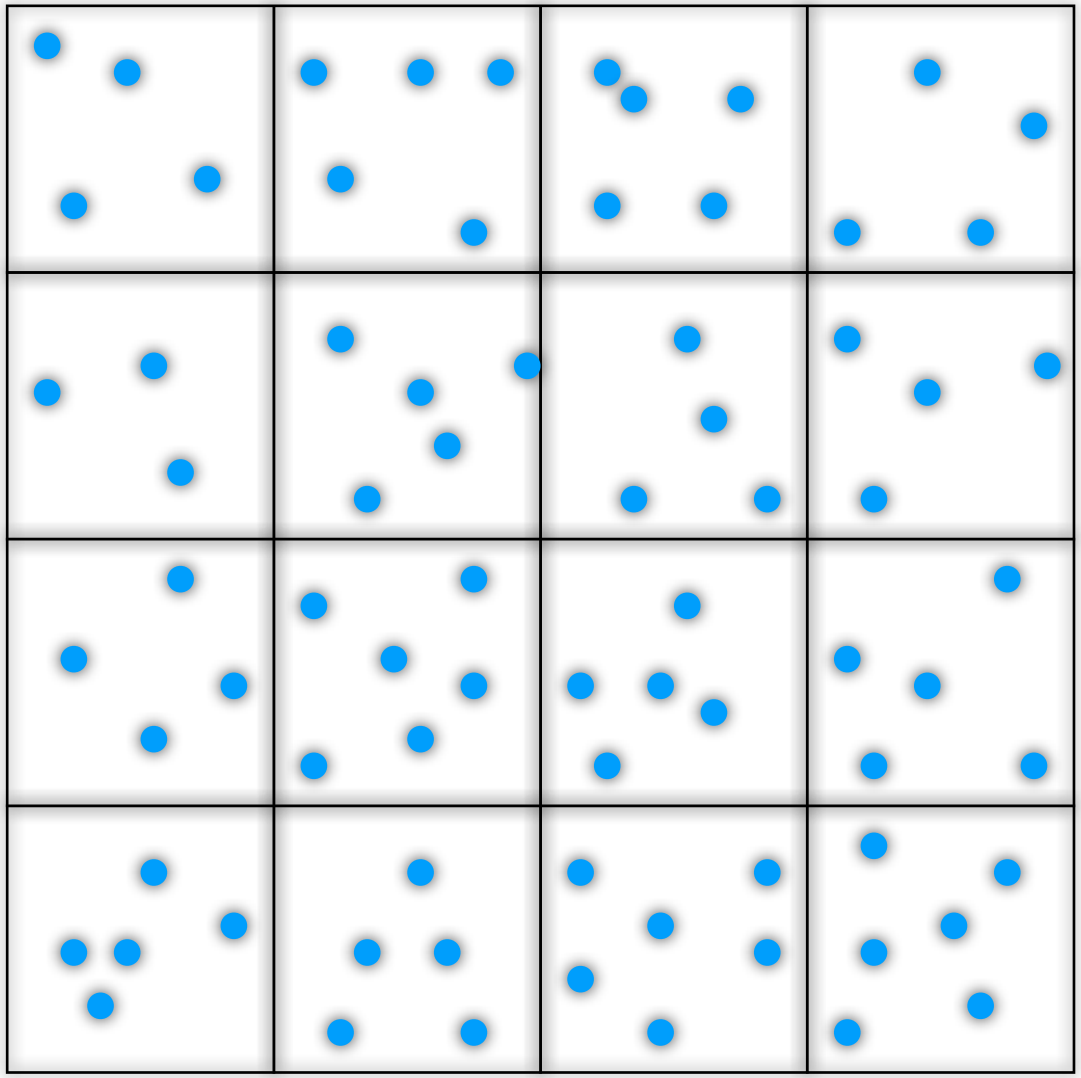

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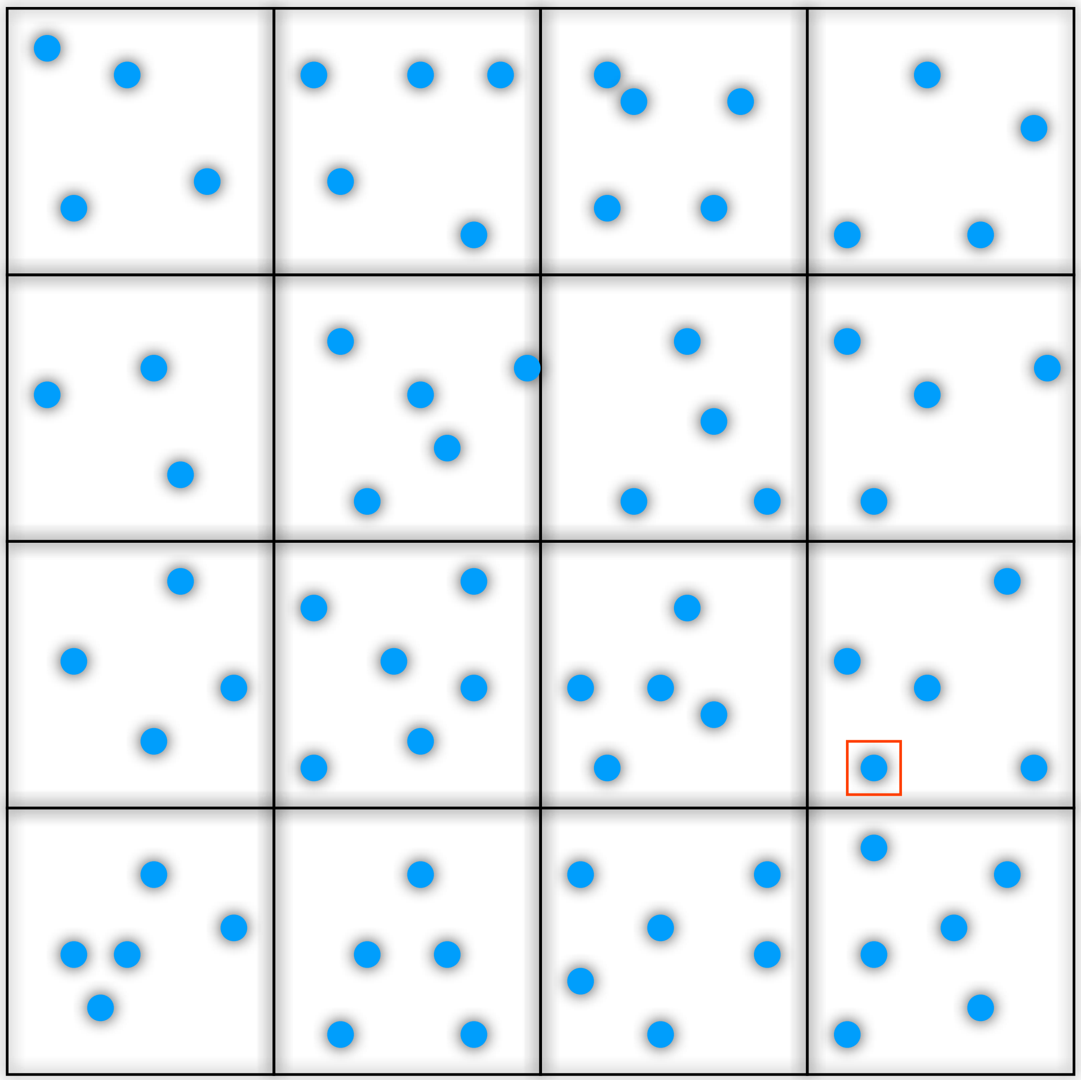

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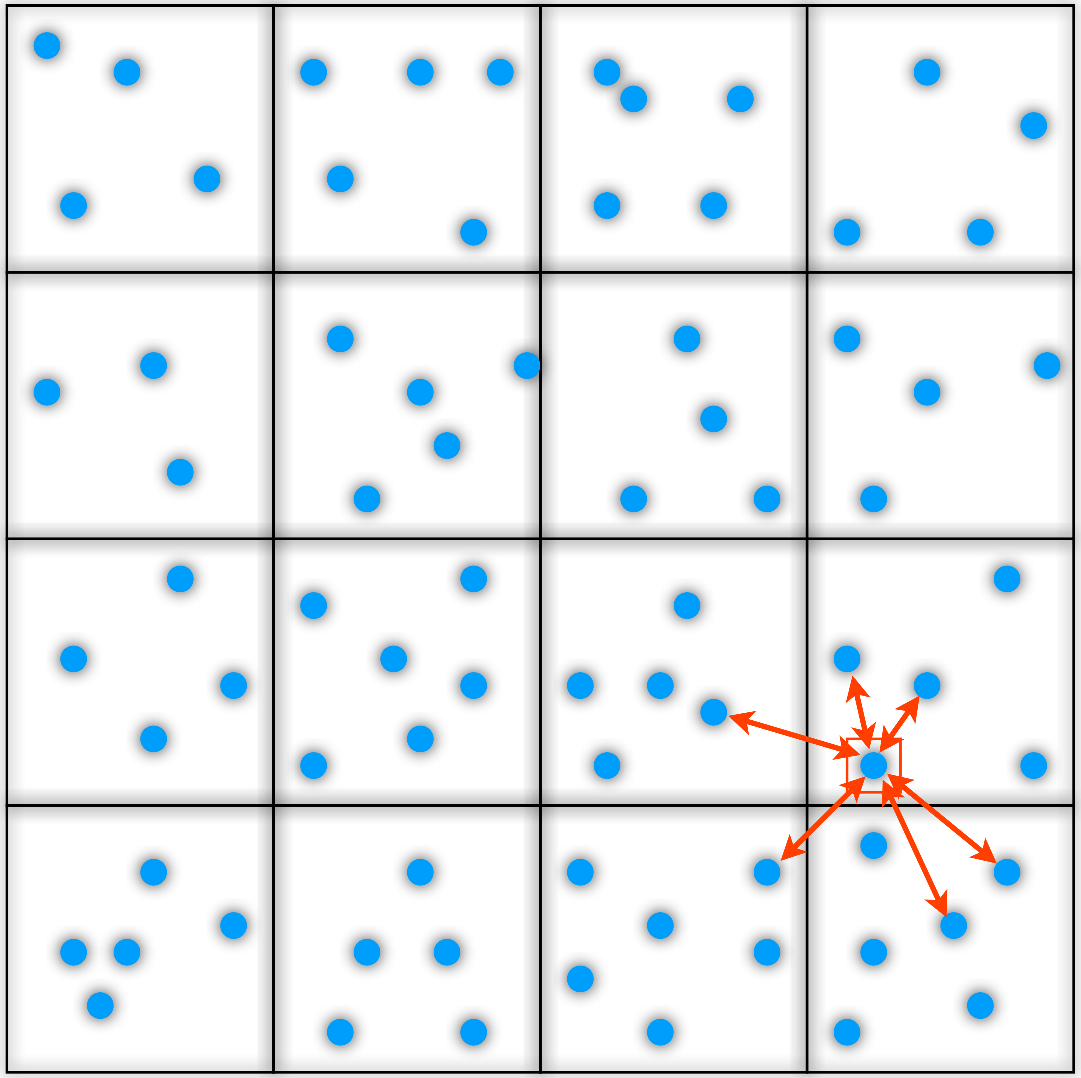

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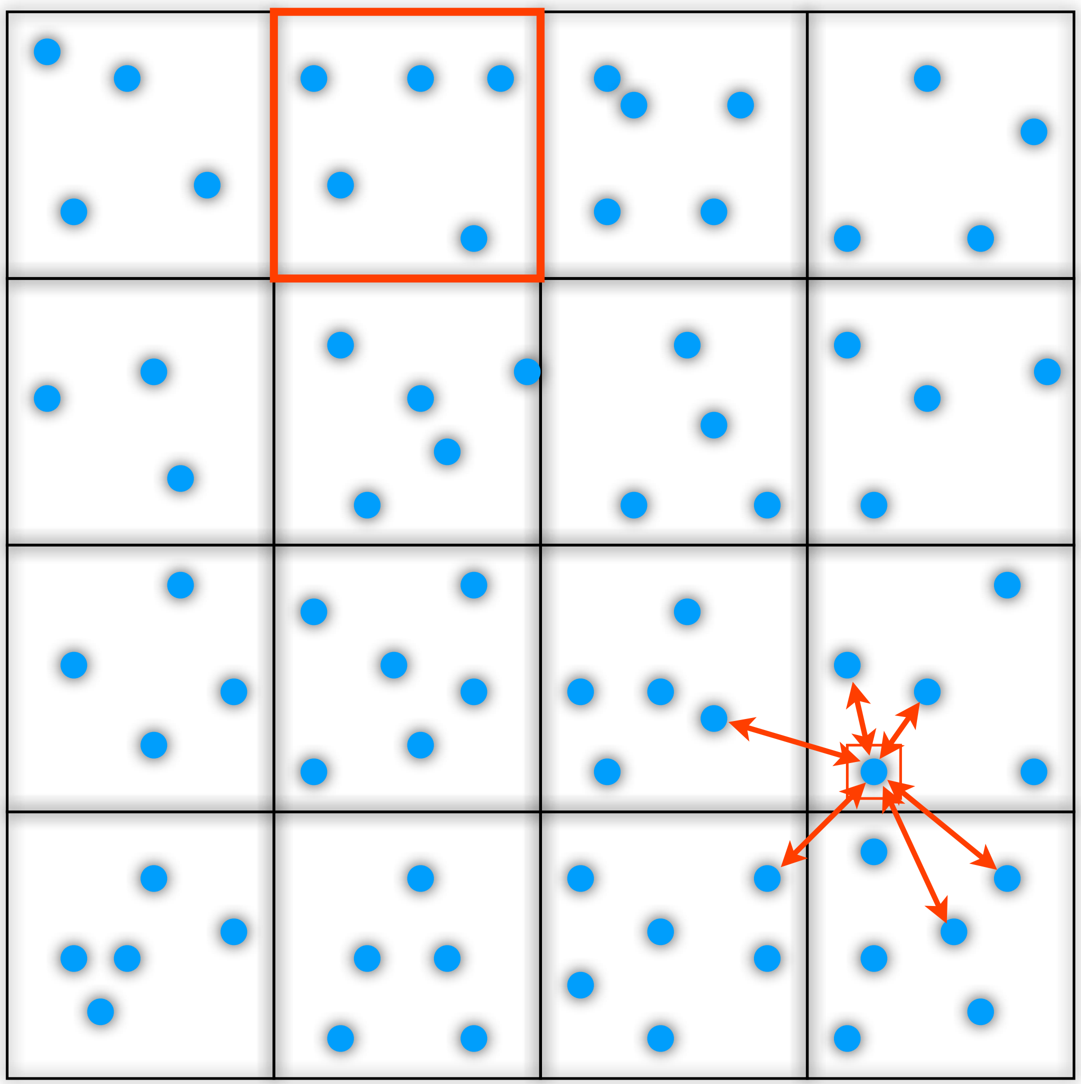

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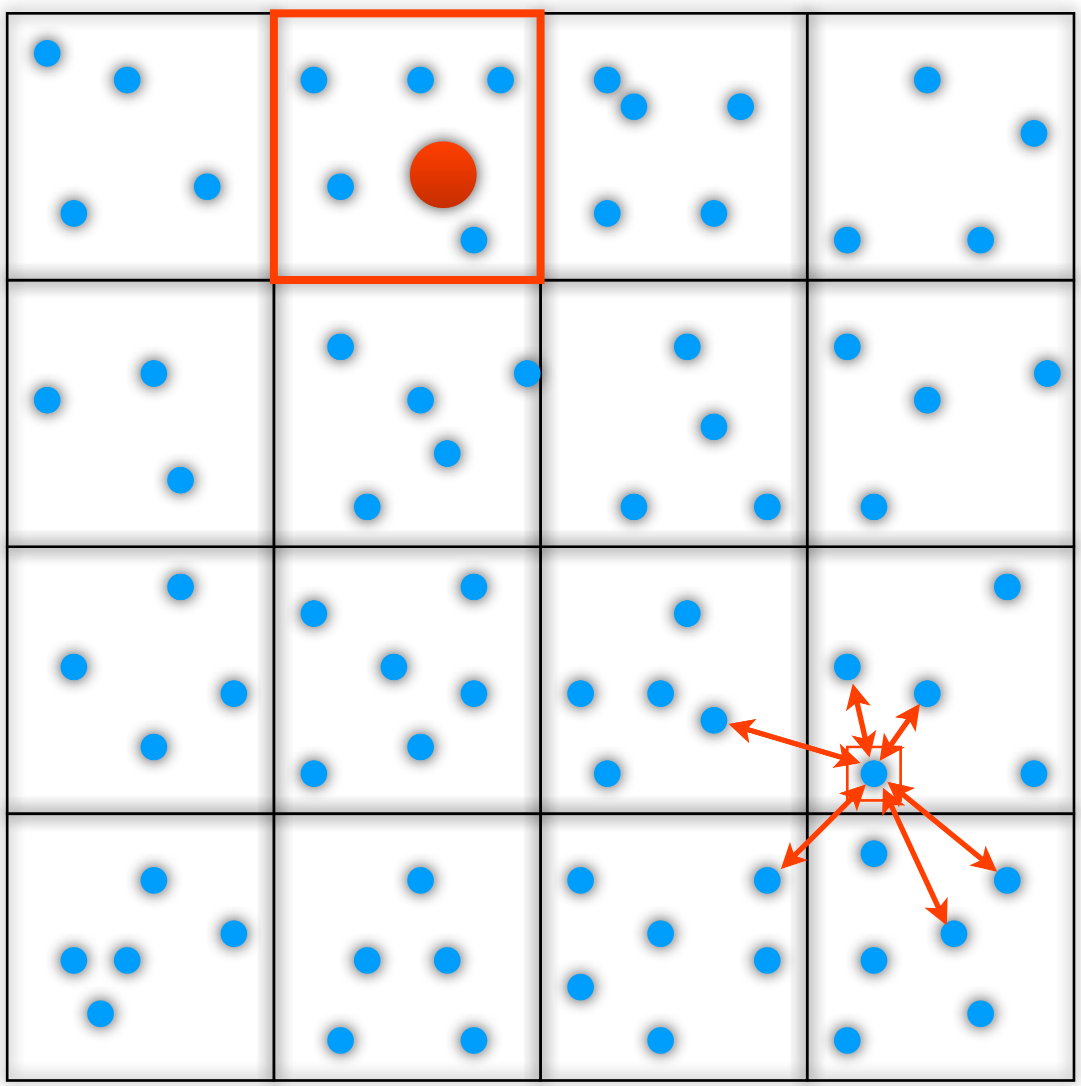

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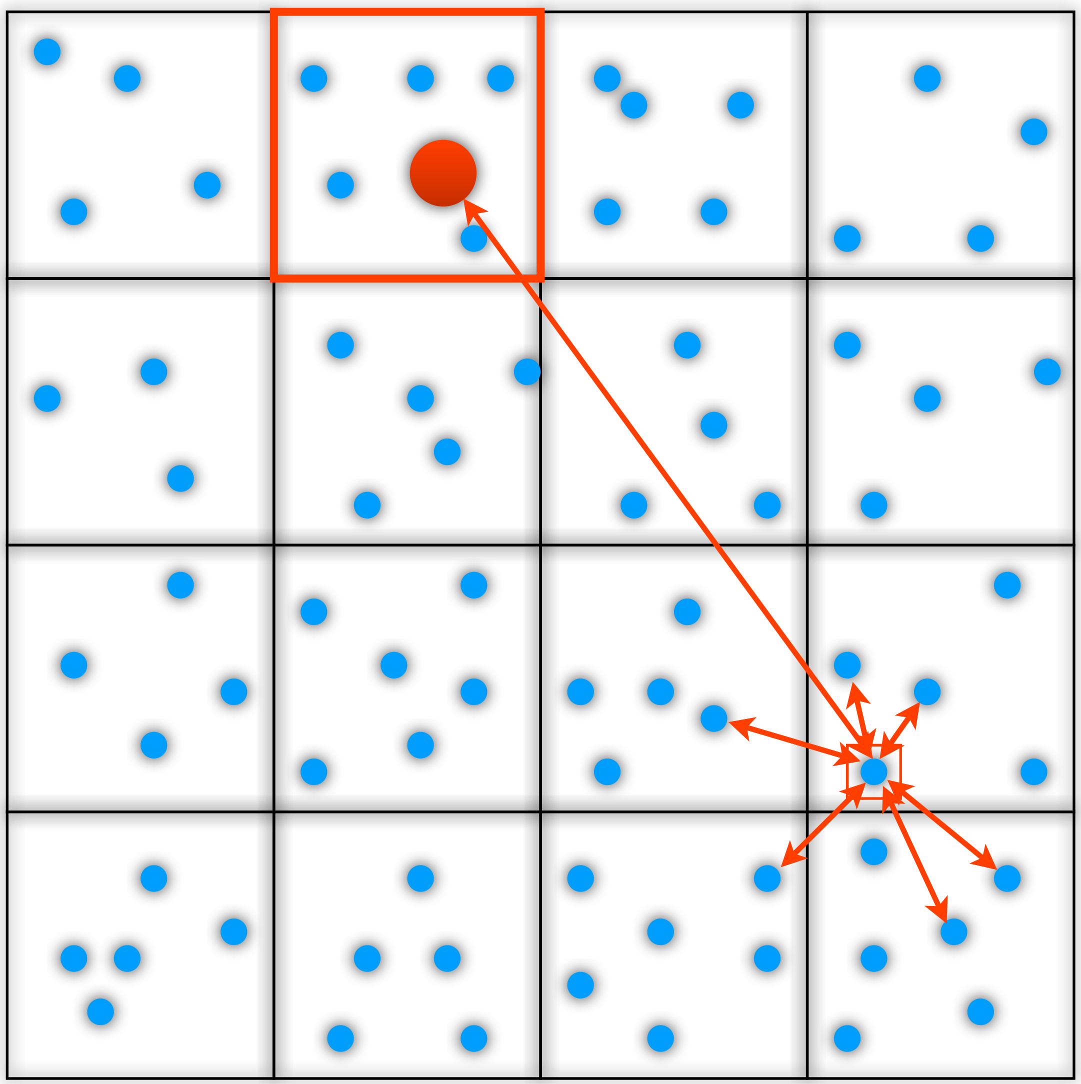

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In interacting DE the coupling determines two different gravitational forces for dark matter and baryons:

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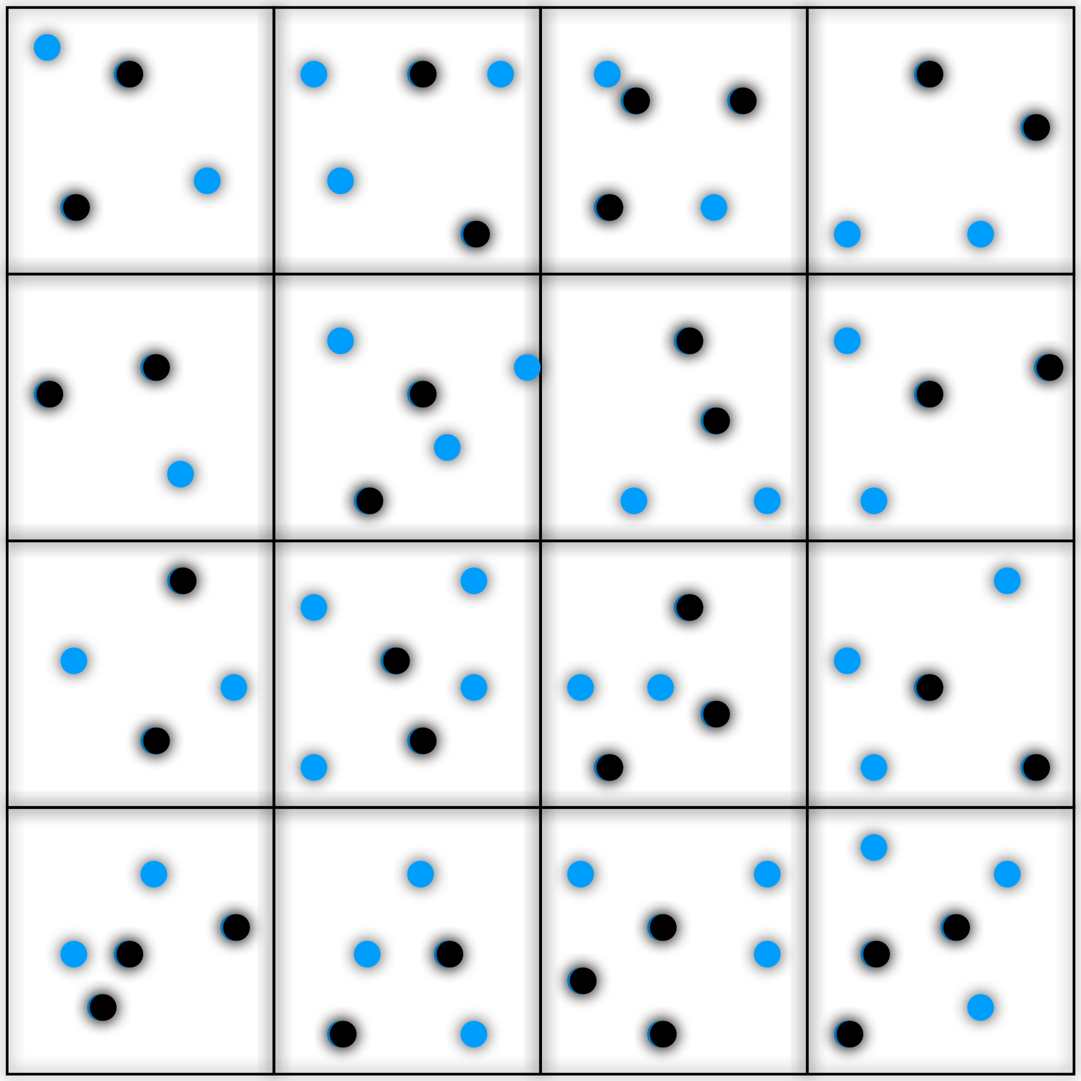

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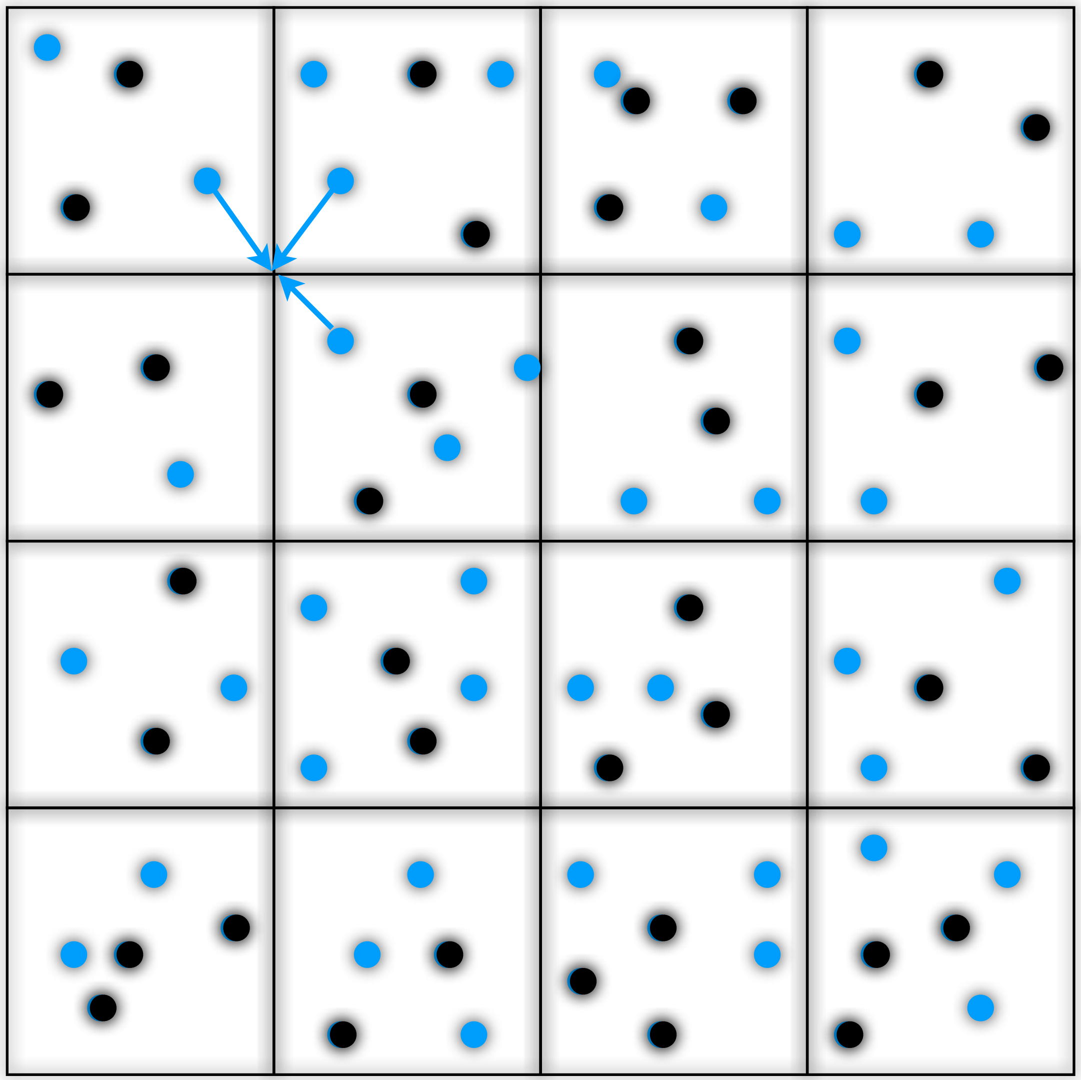

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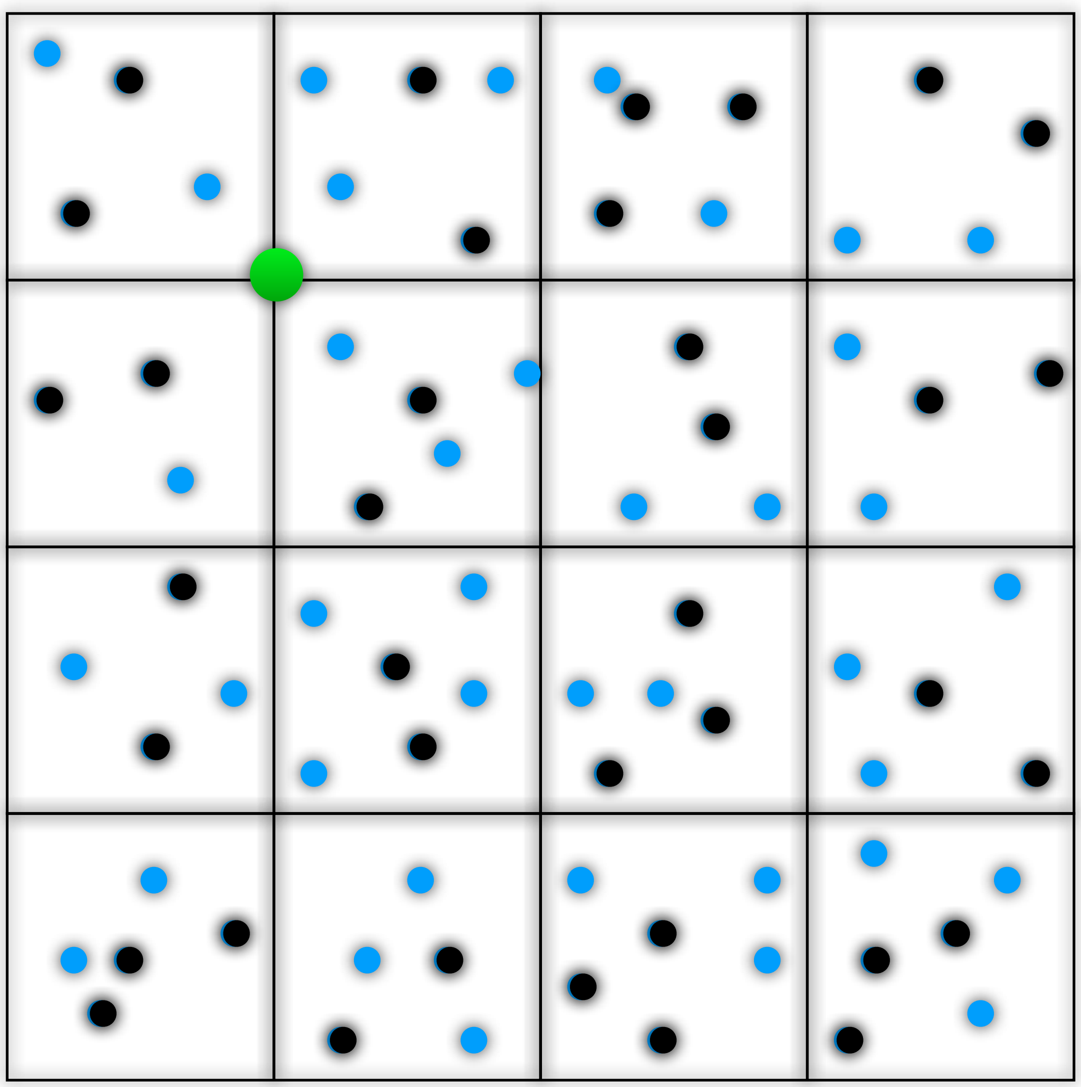

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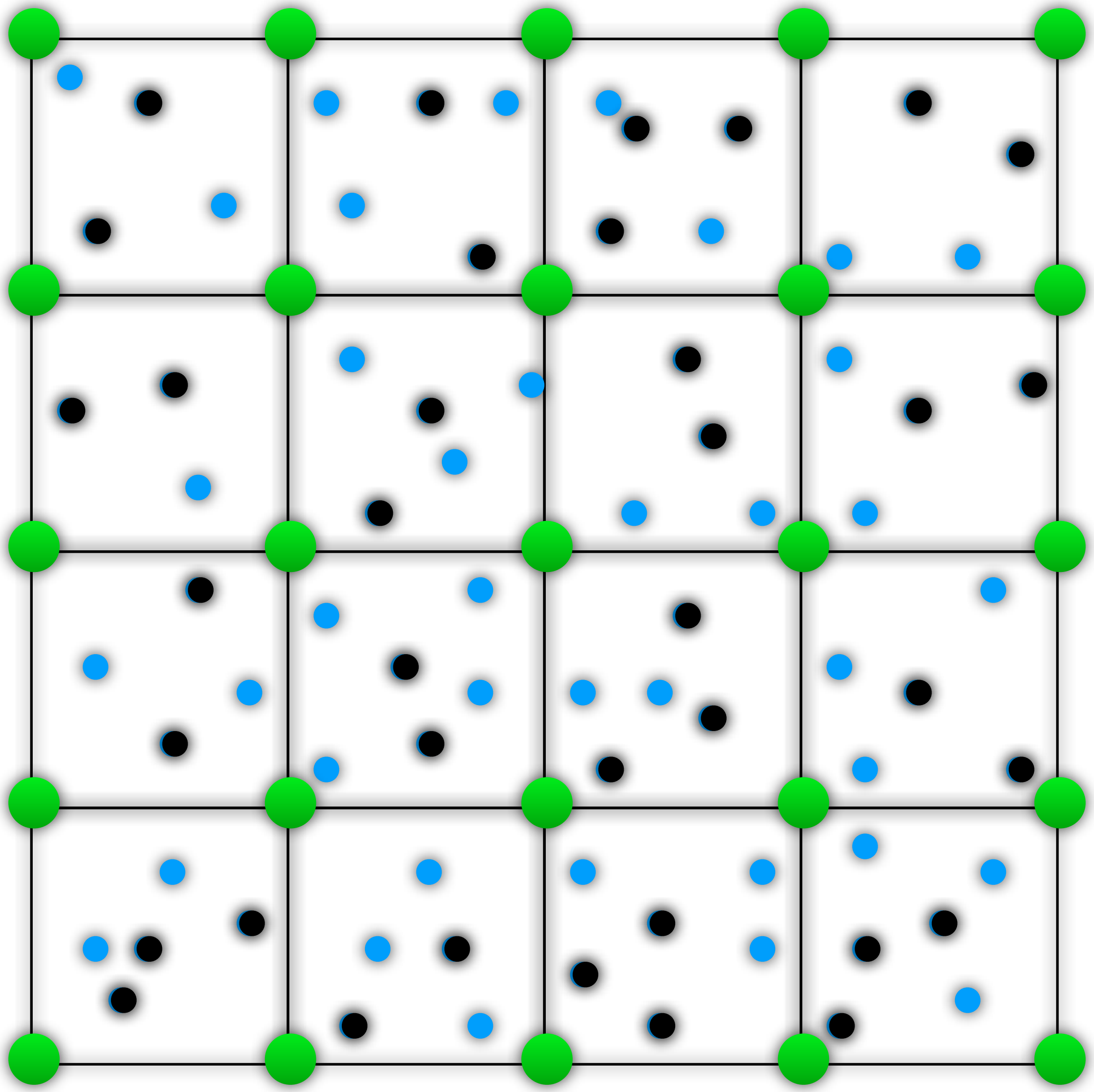

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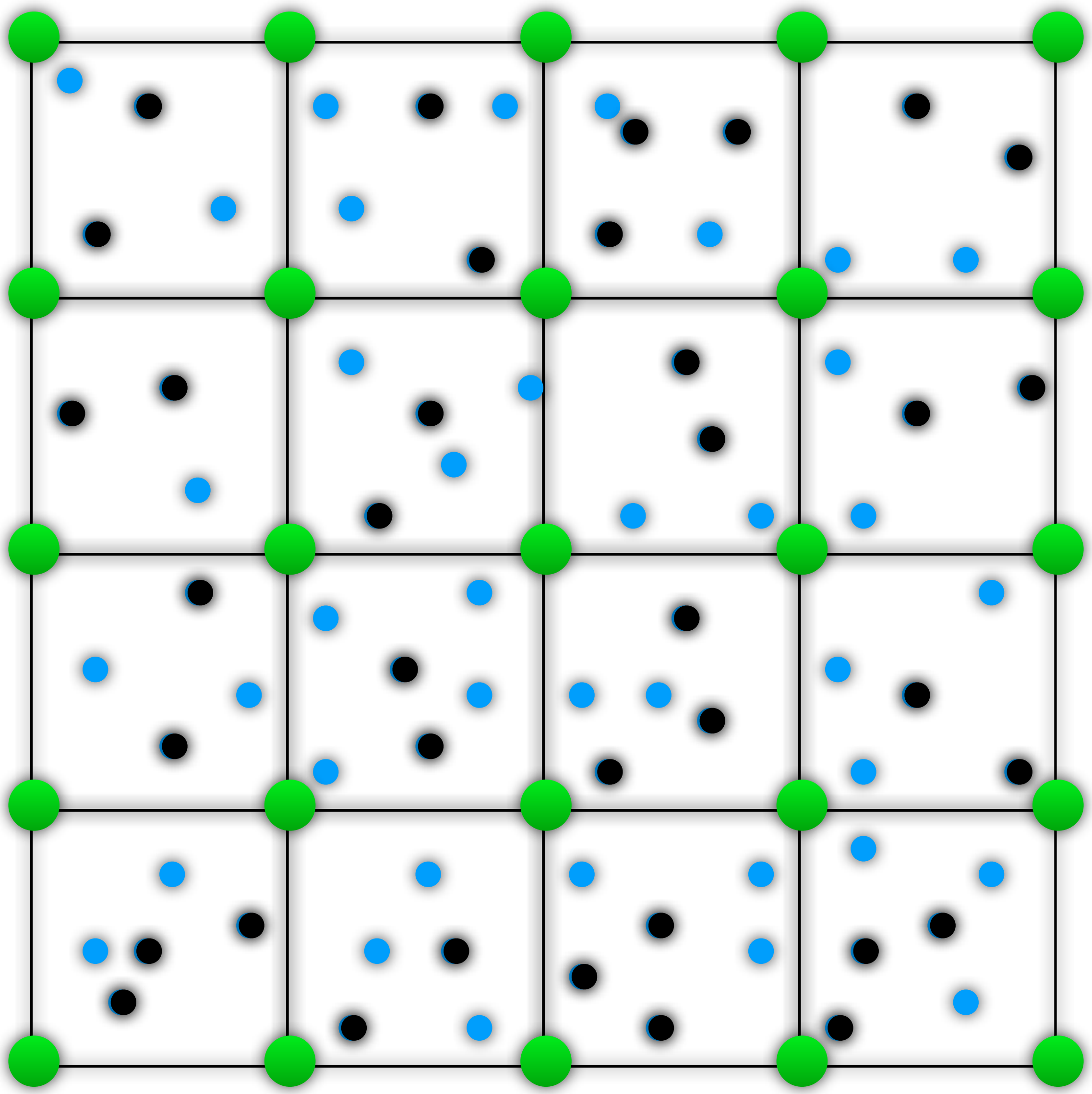

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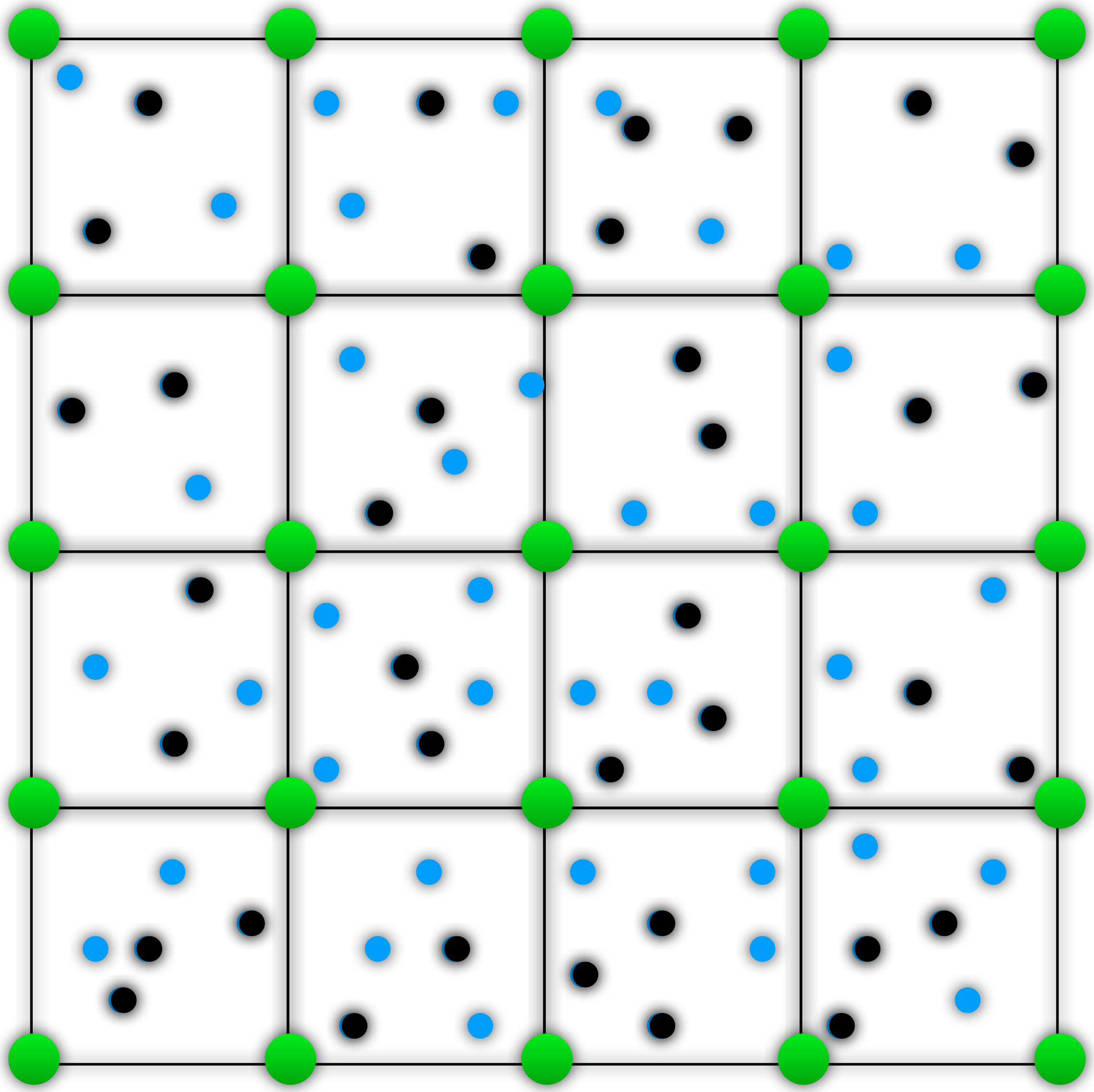

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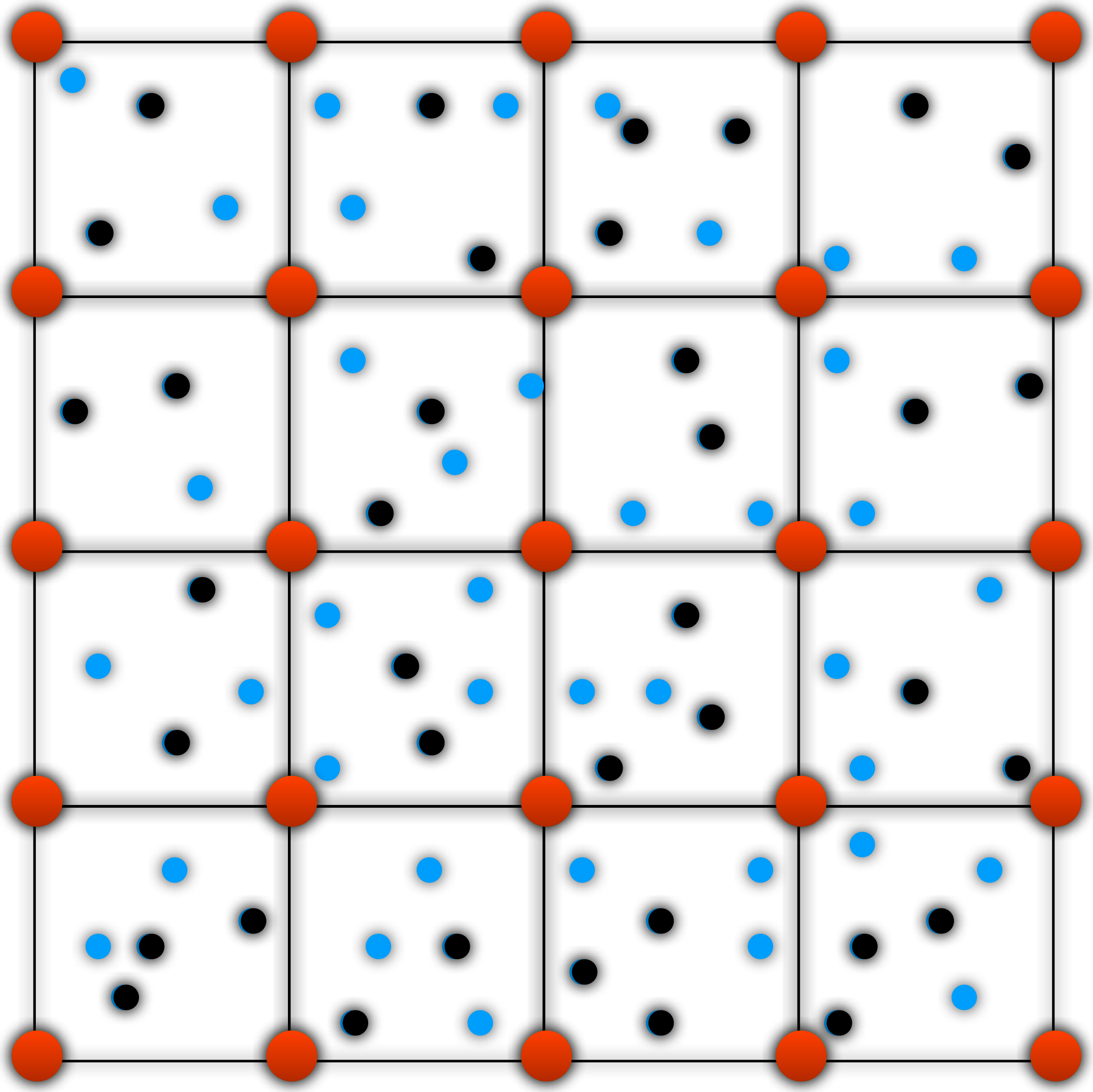

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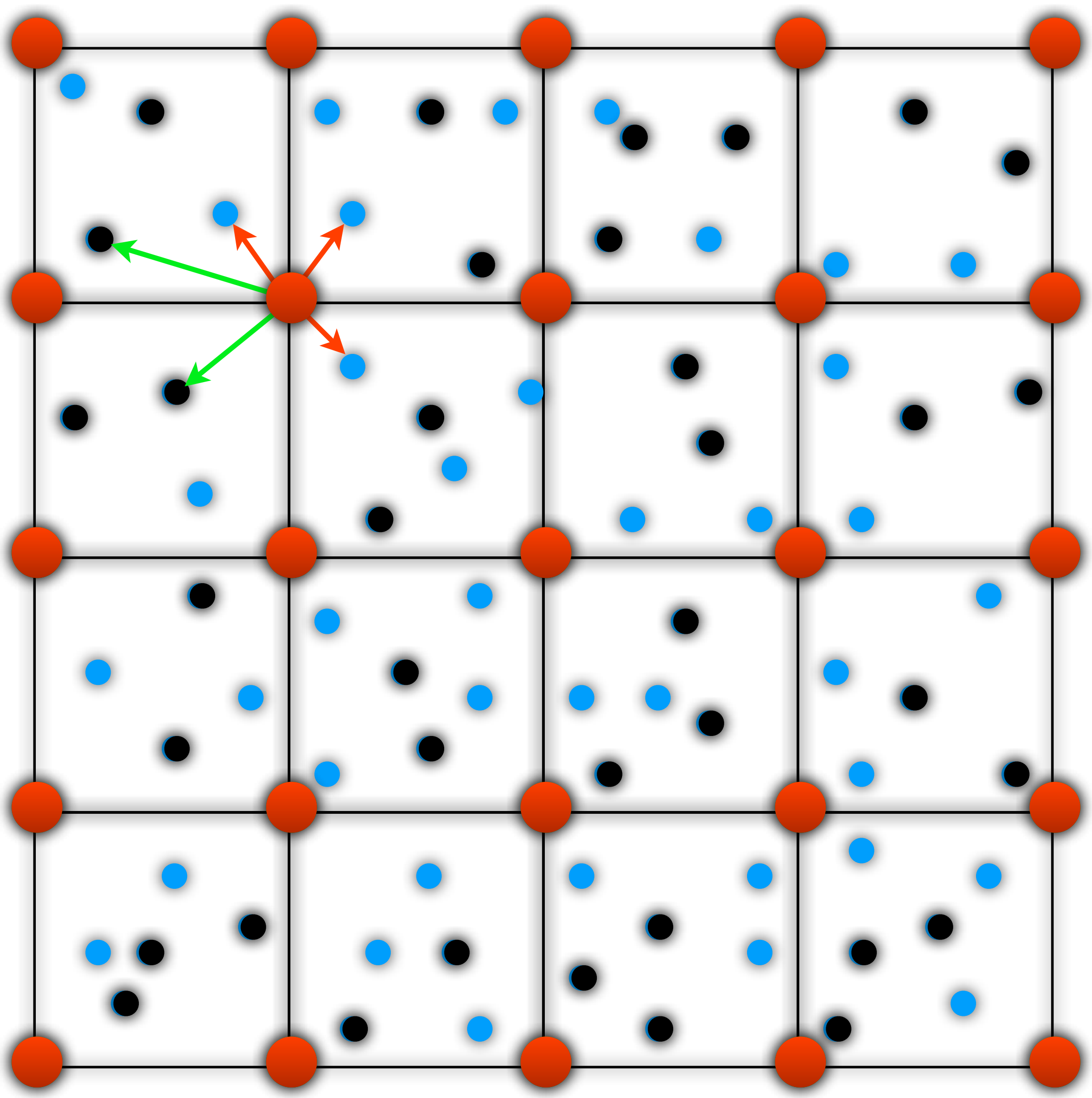

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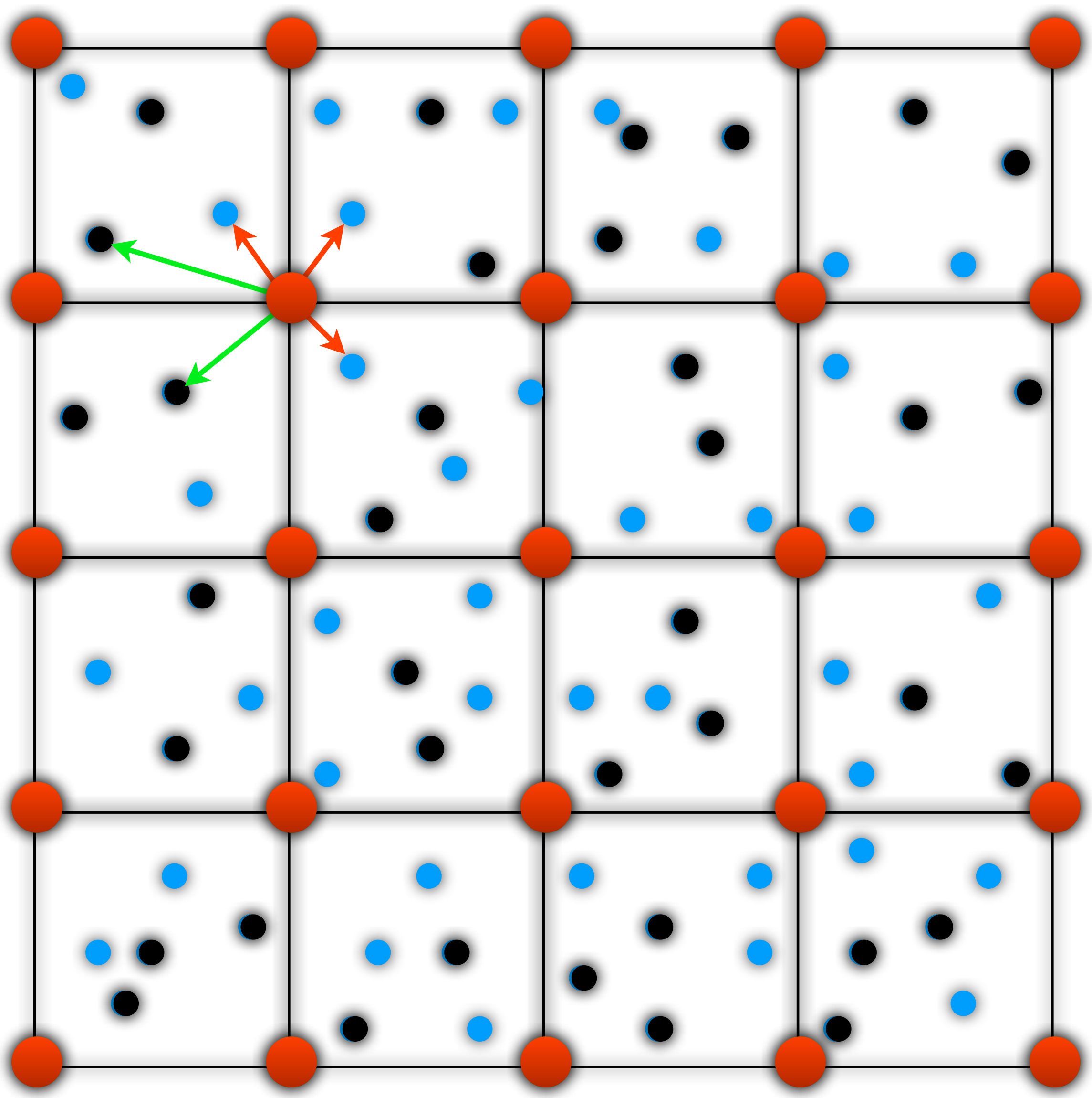

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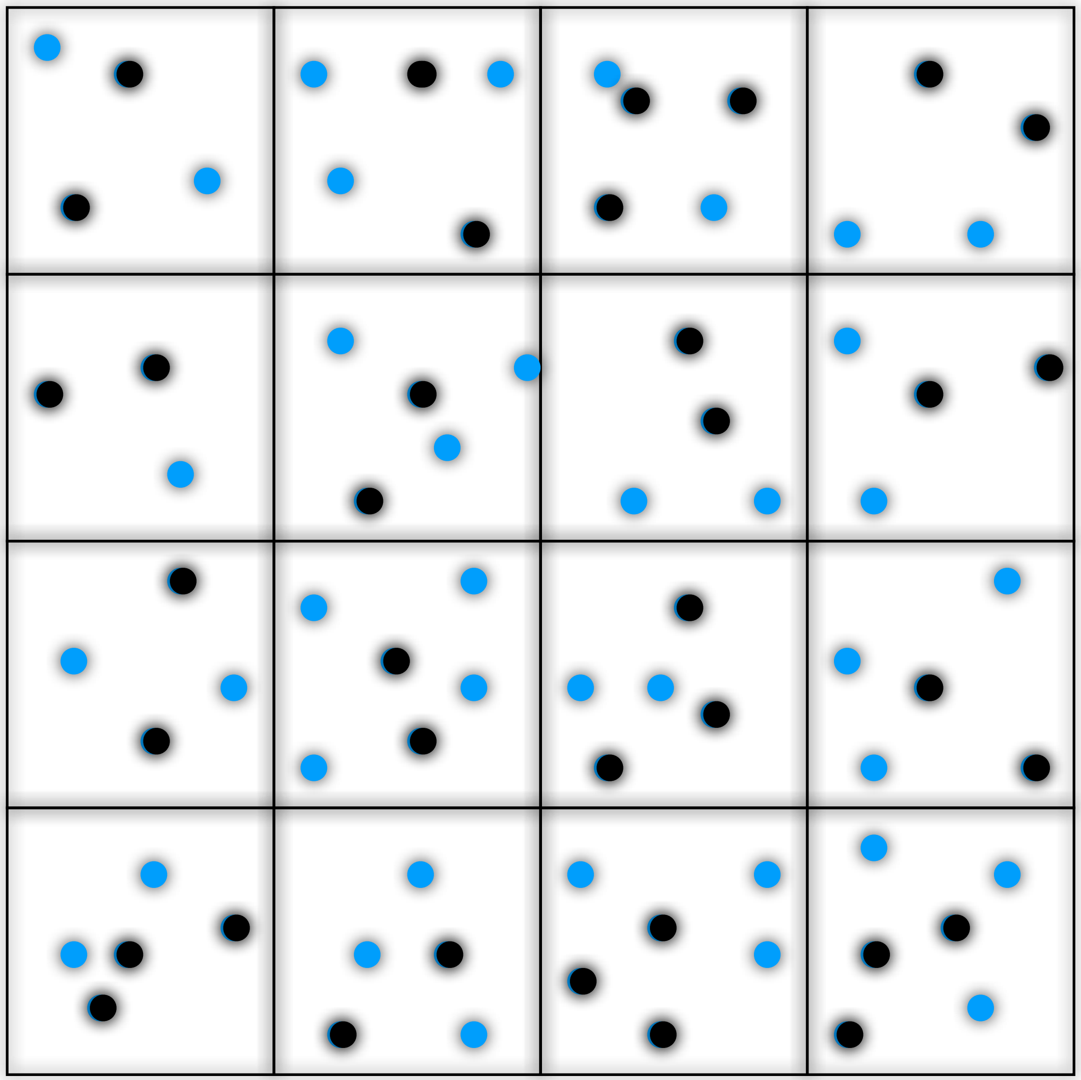

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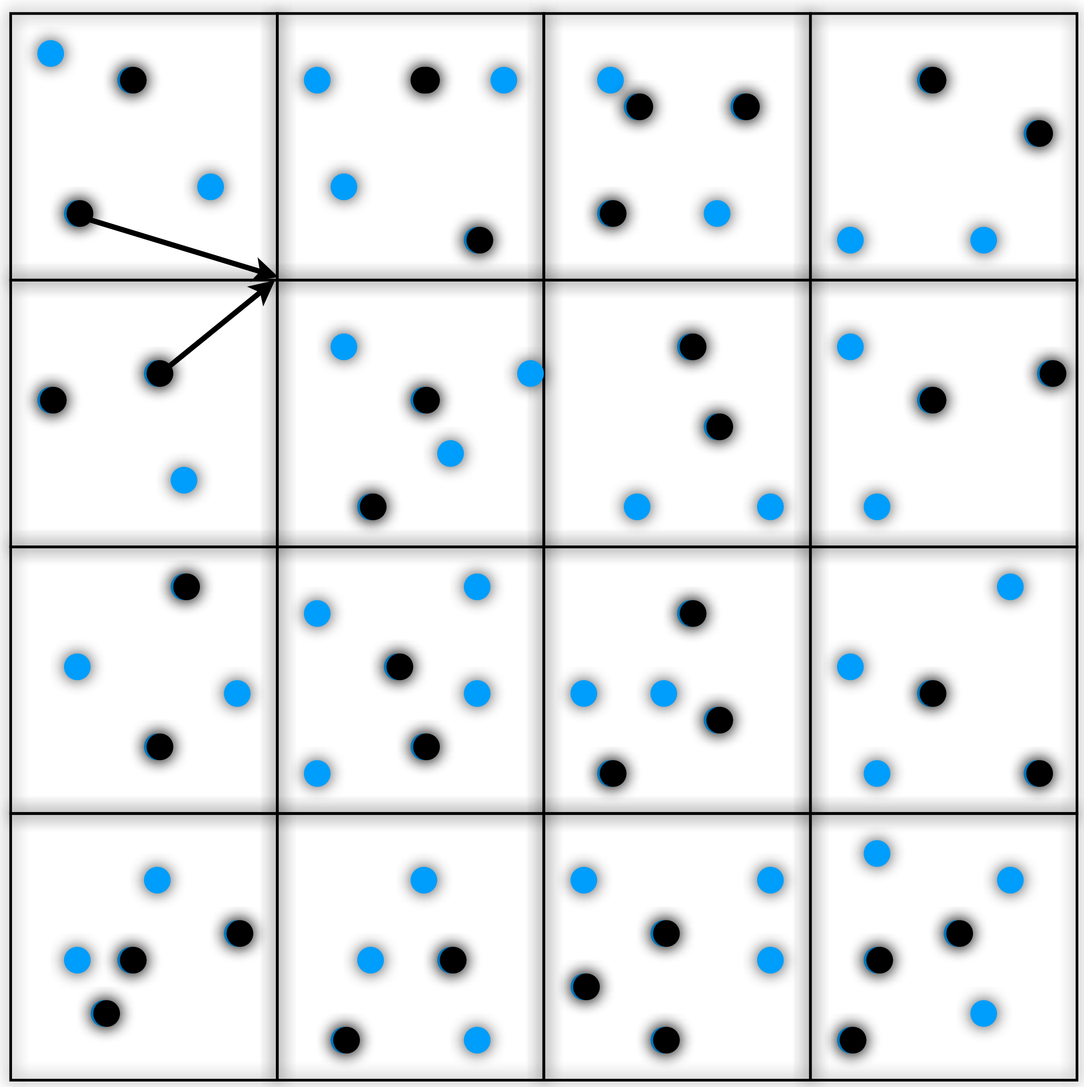

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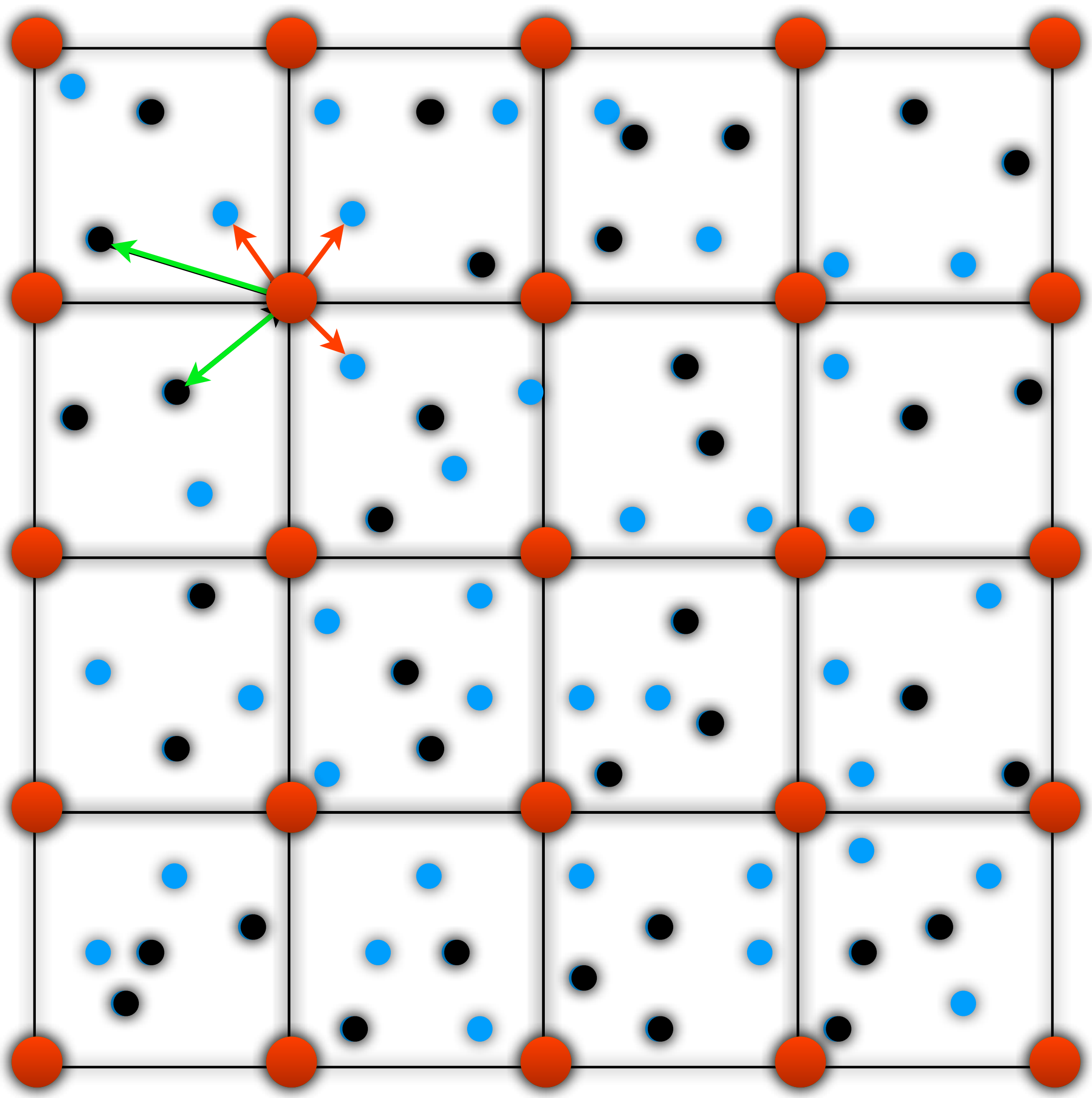

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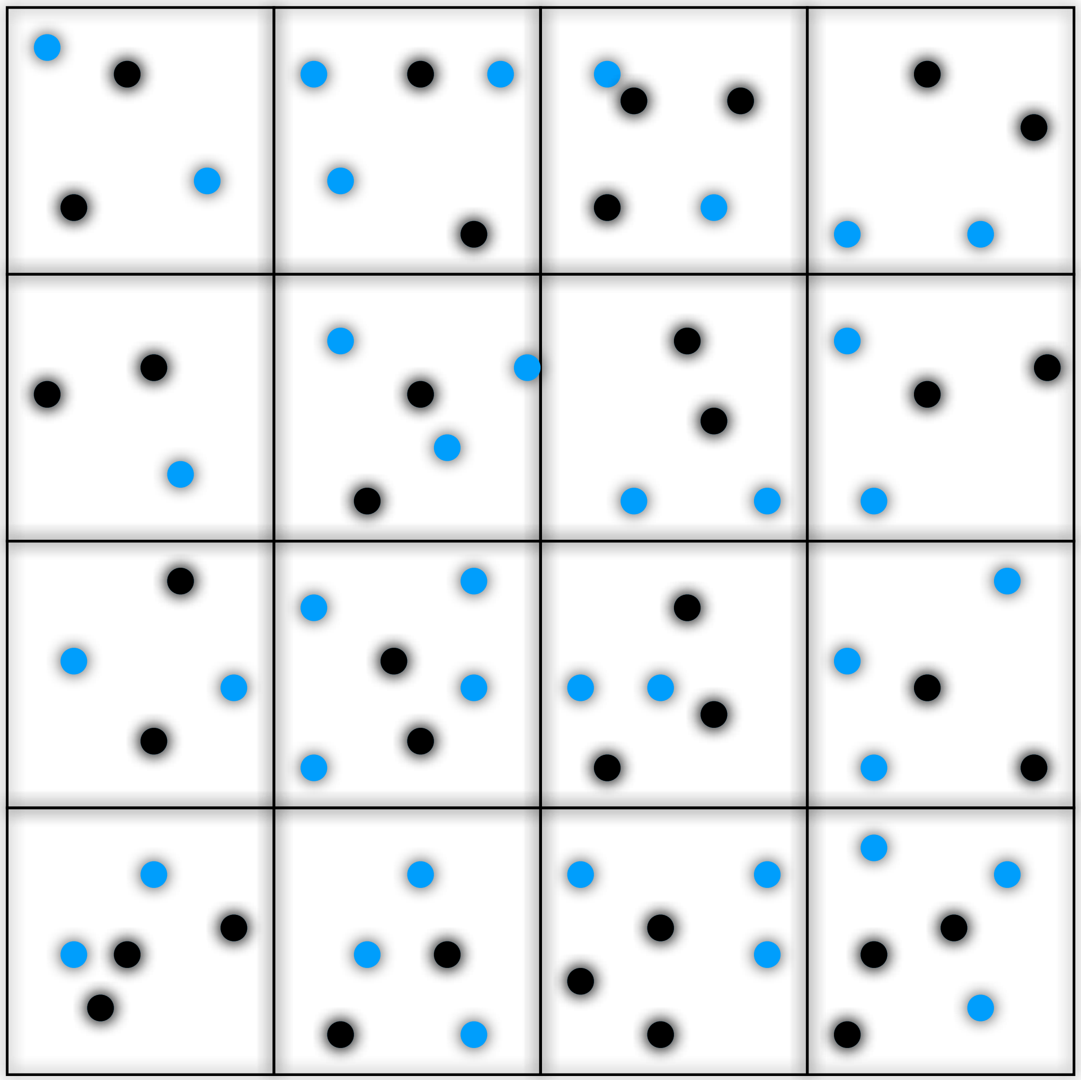

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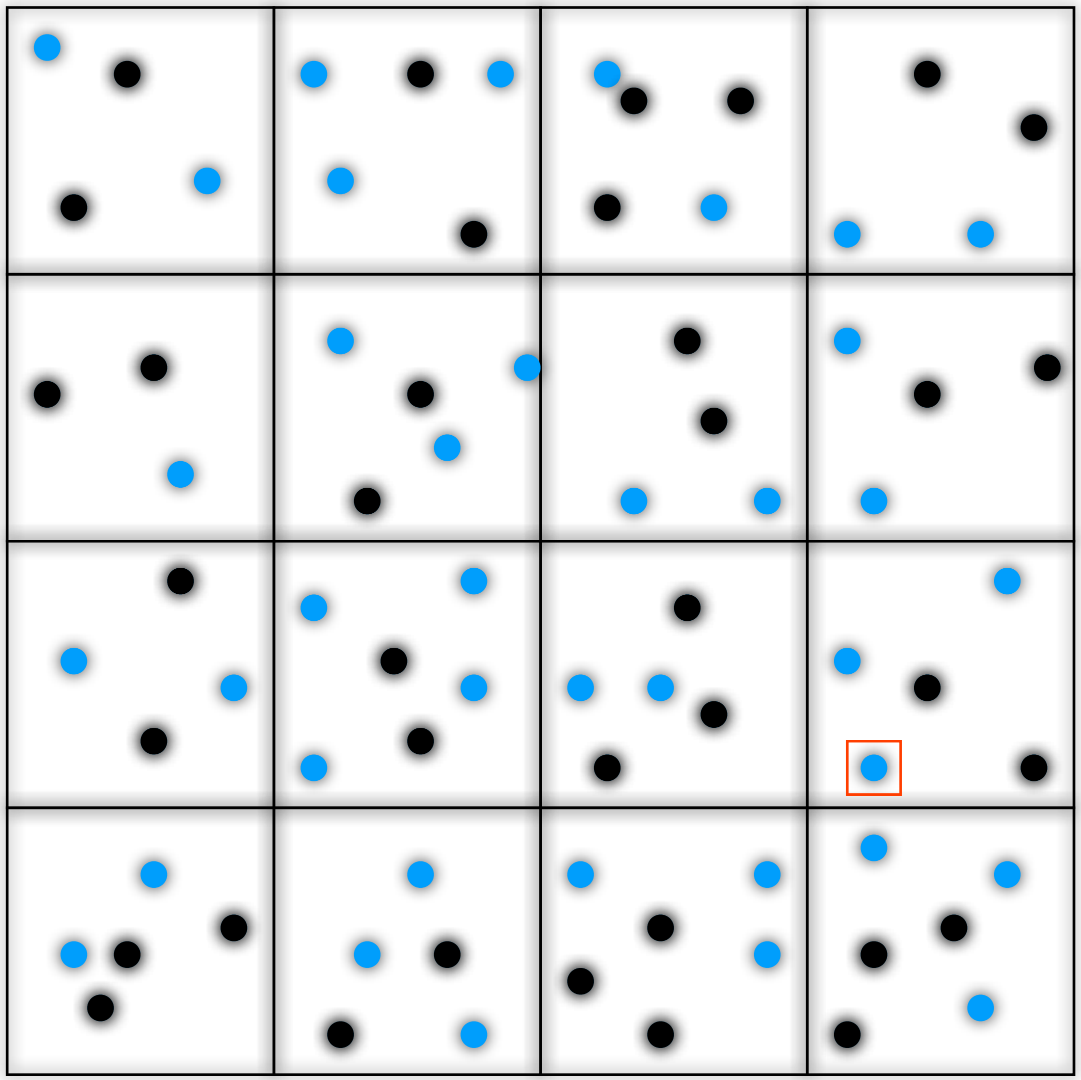

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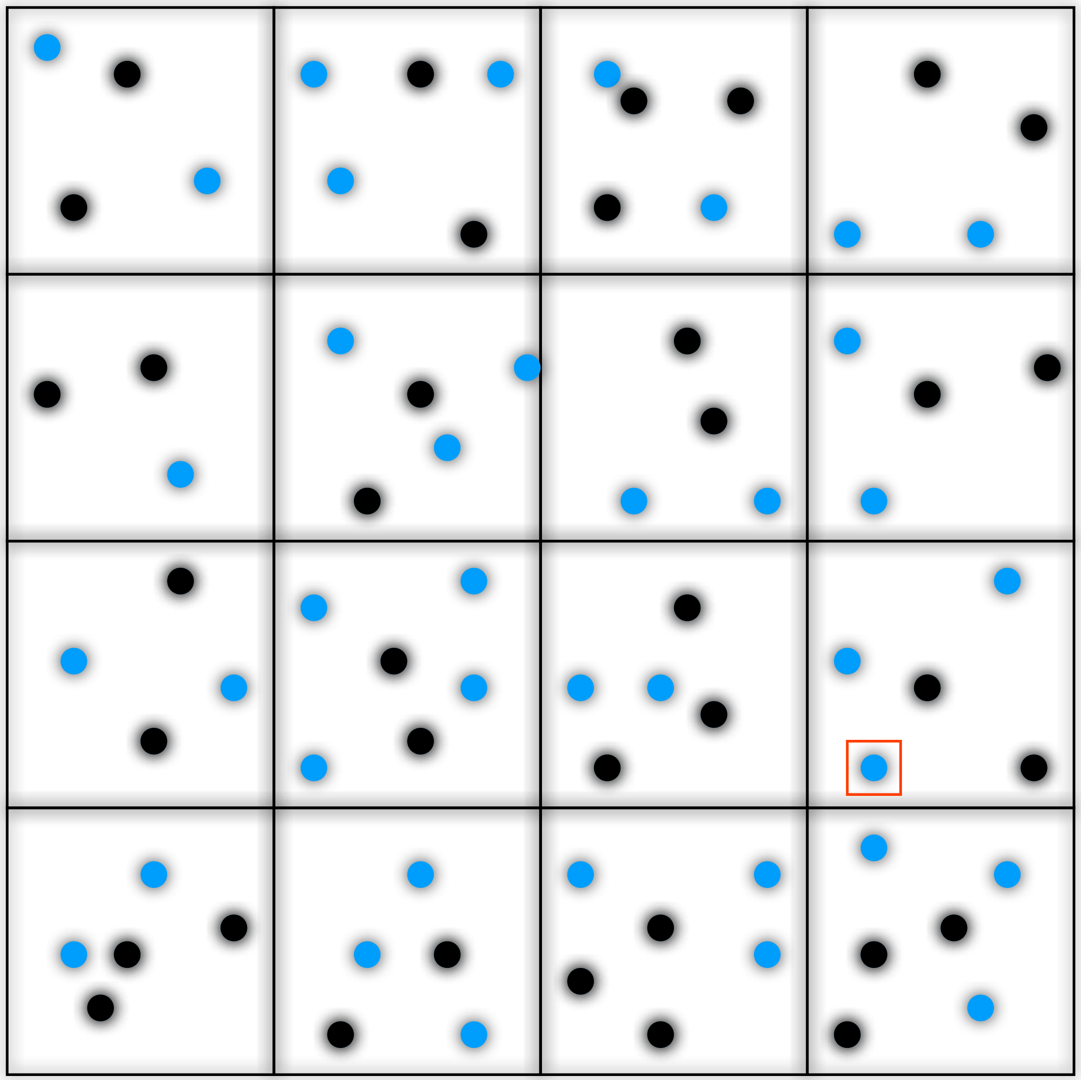

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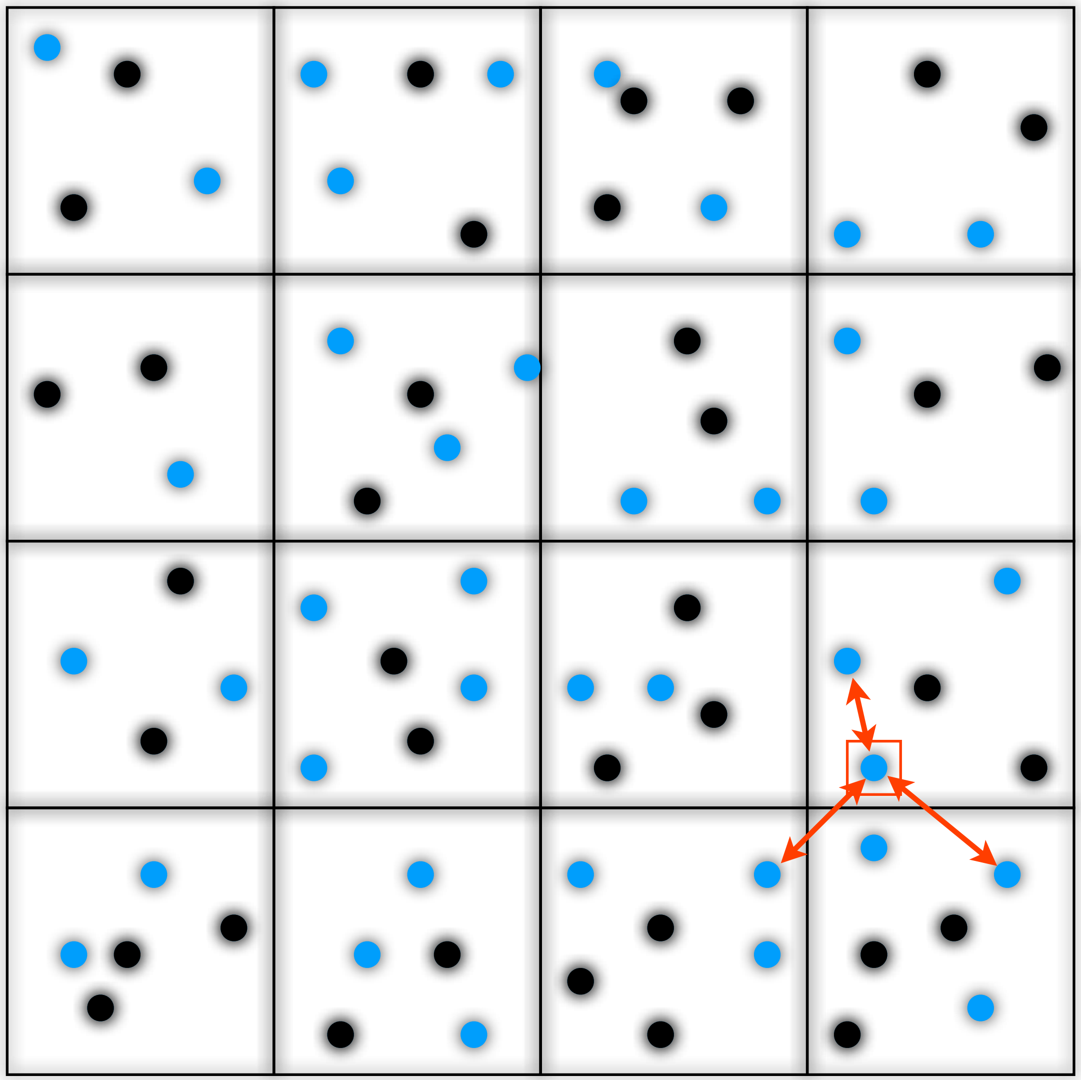

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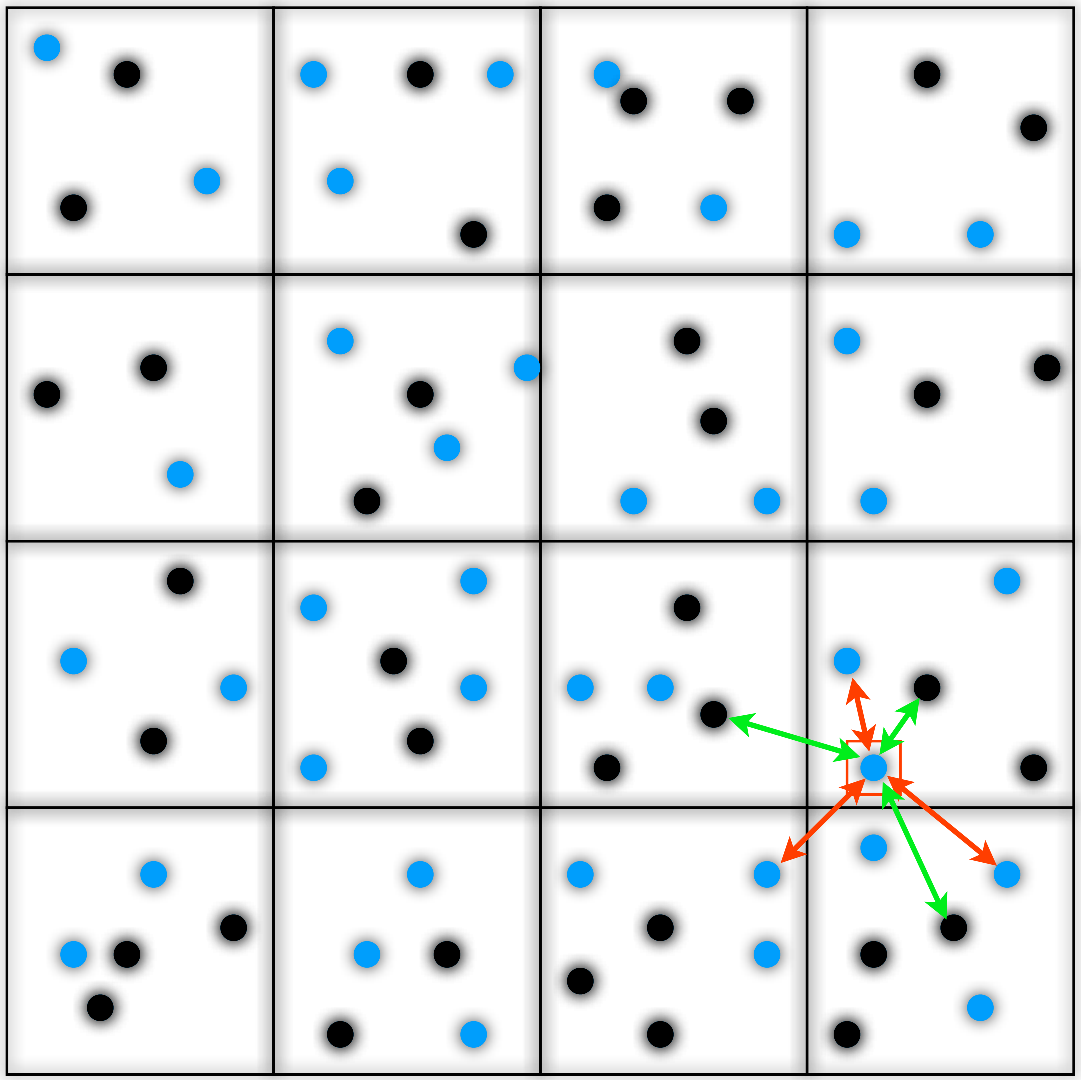

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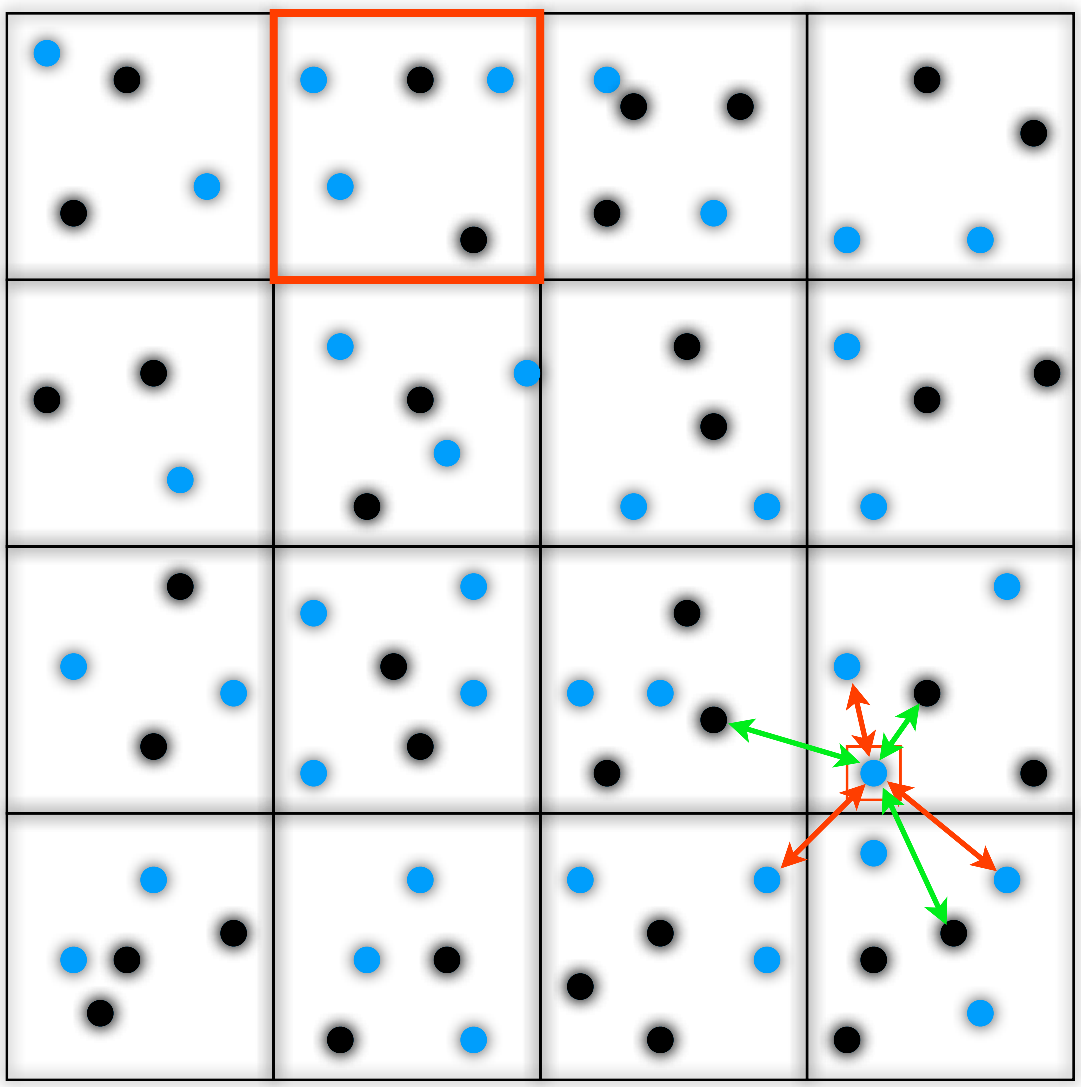

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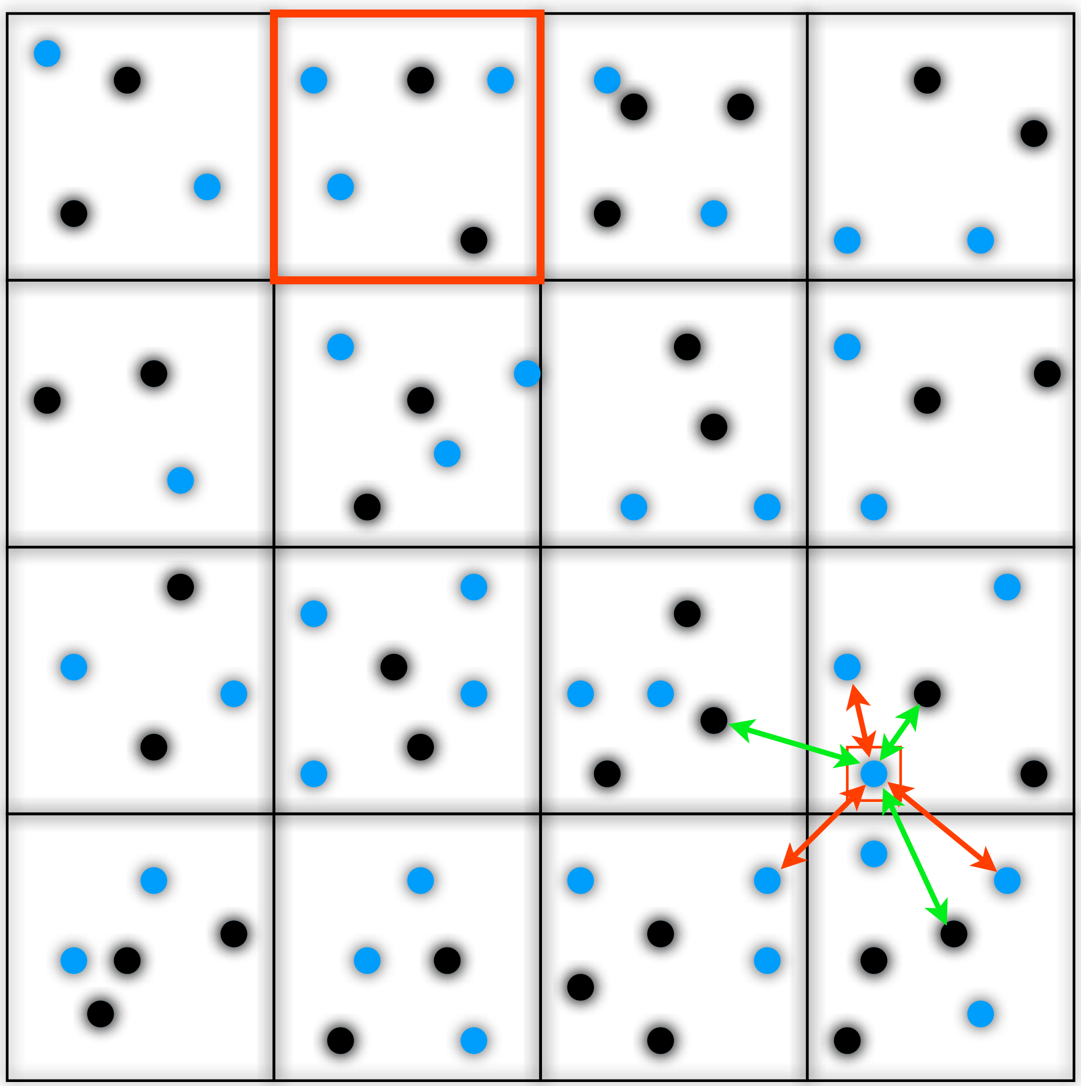

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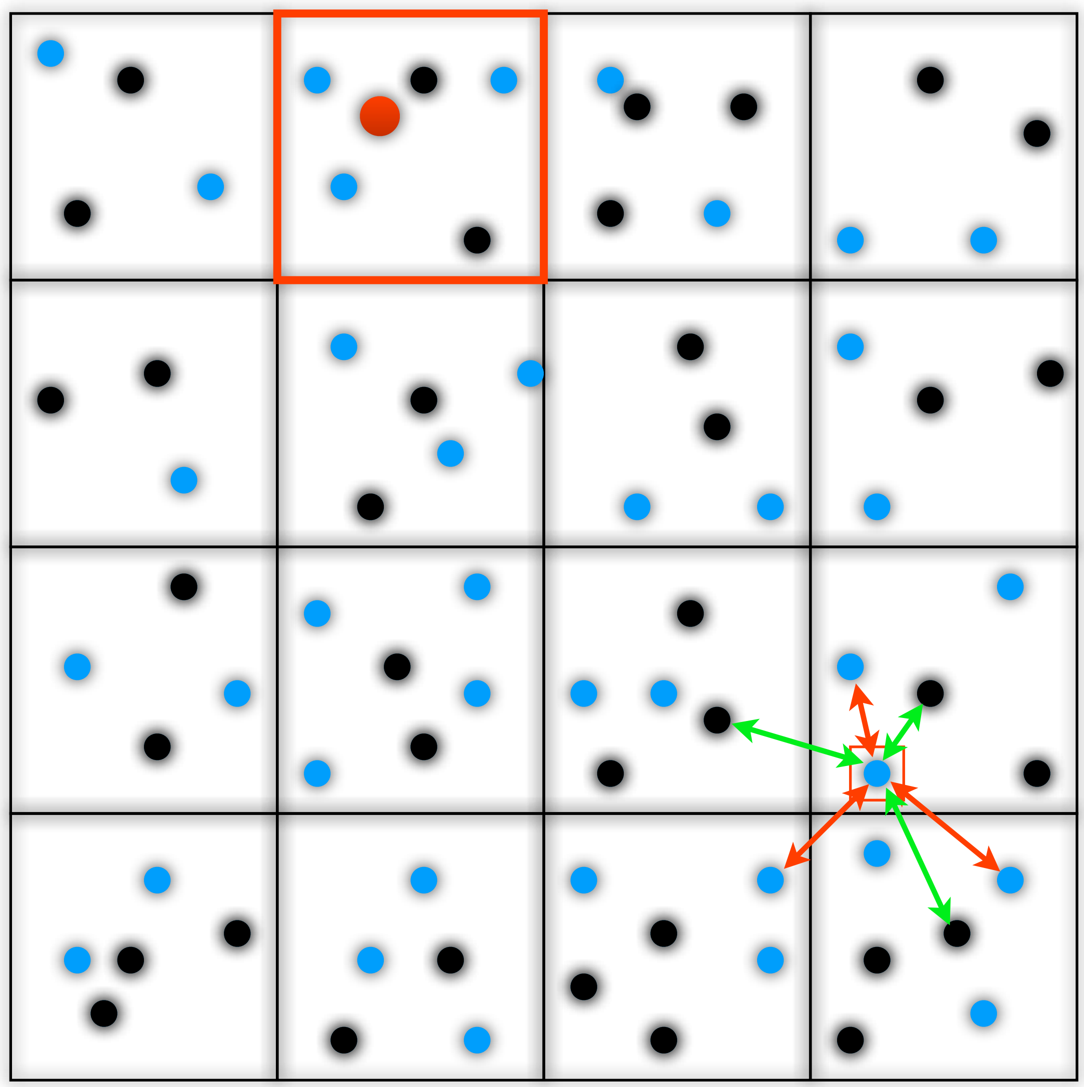

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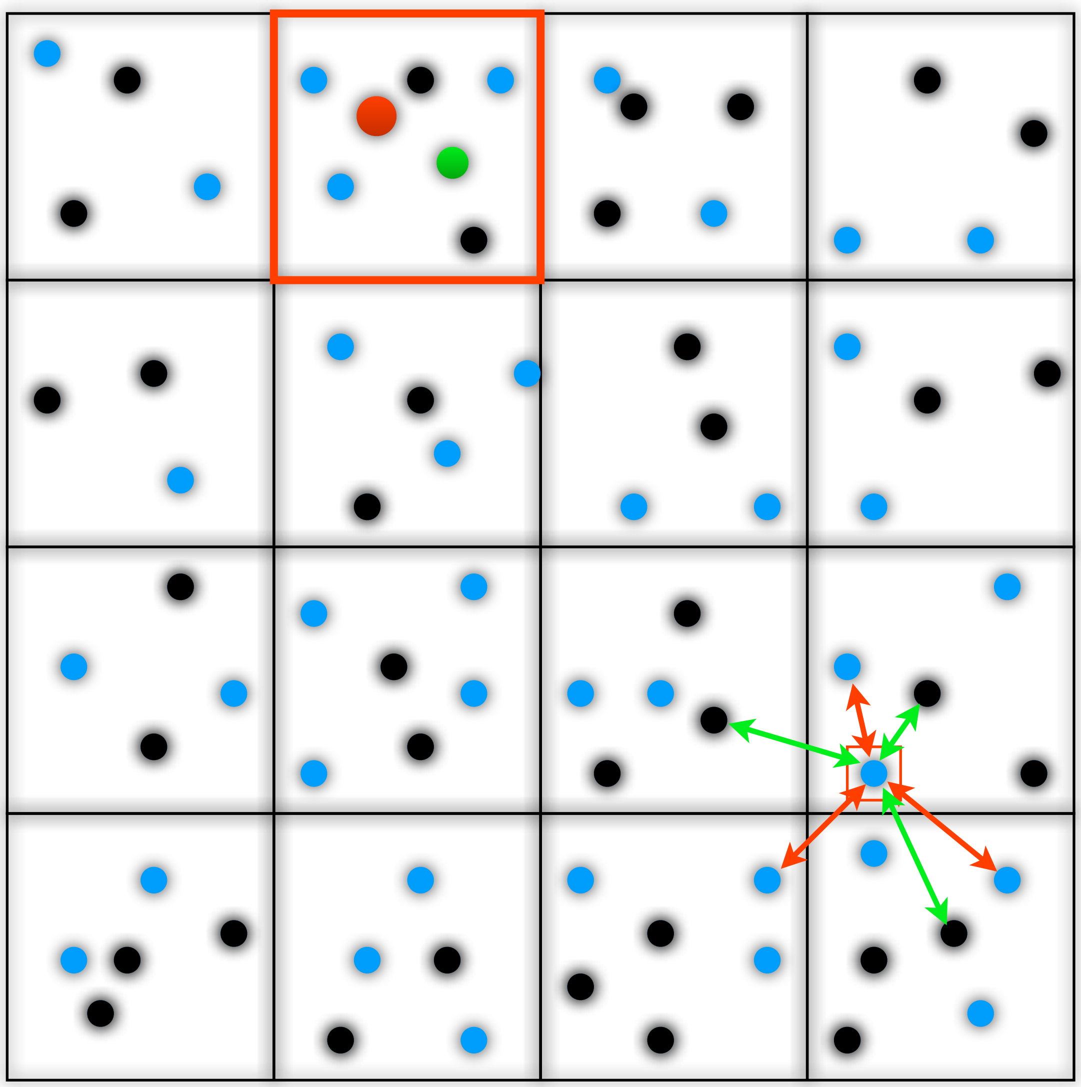

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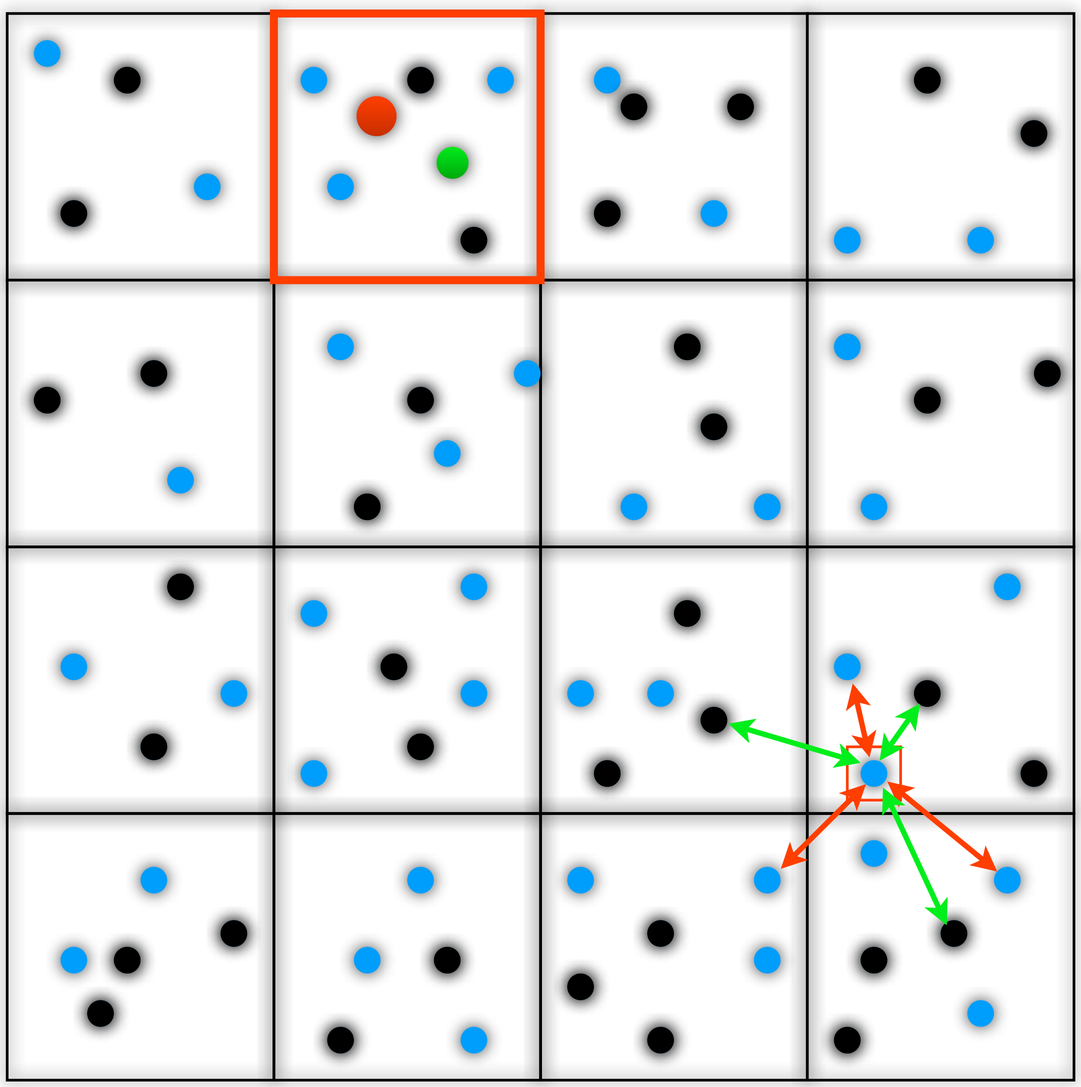

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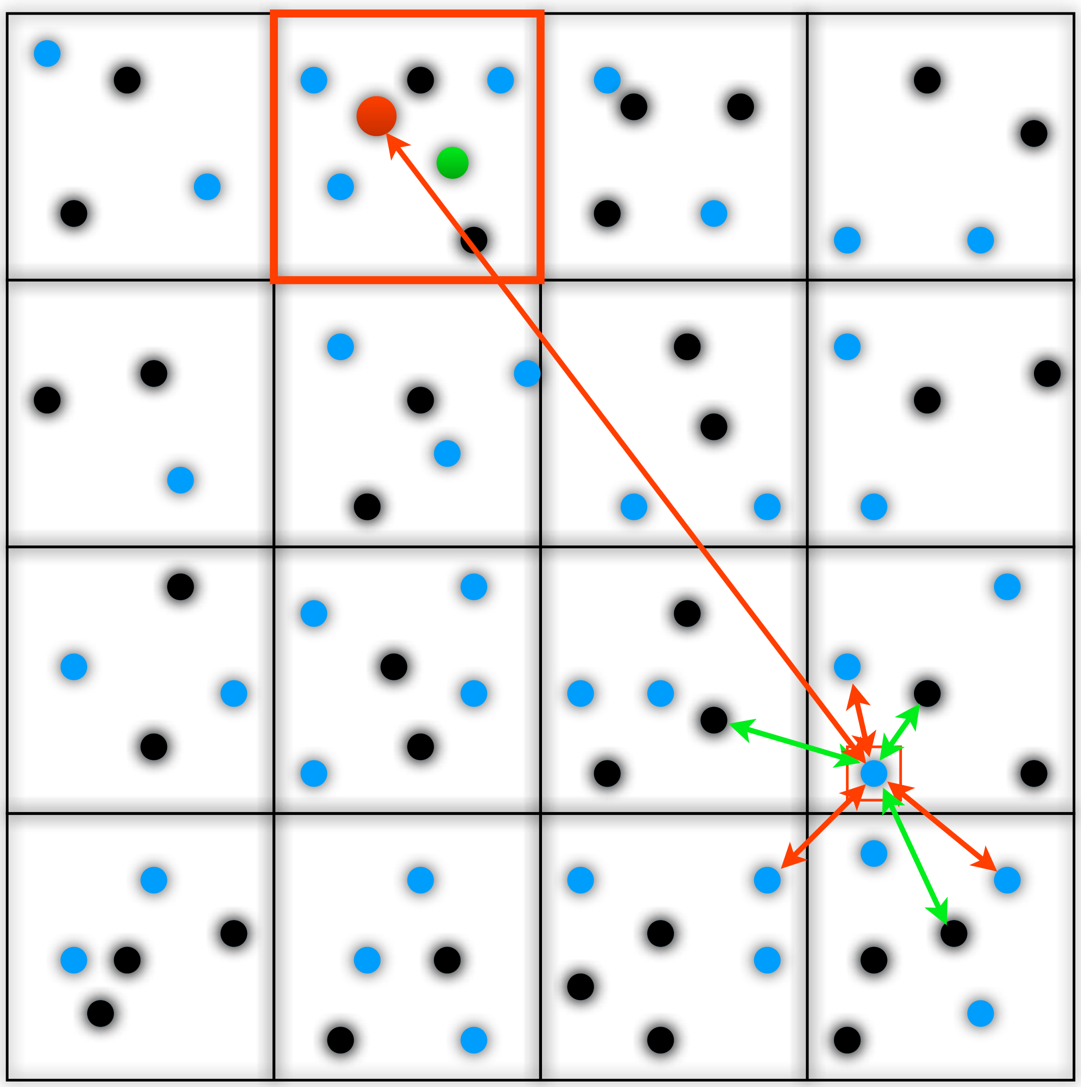

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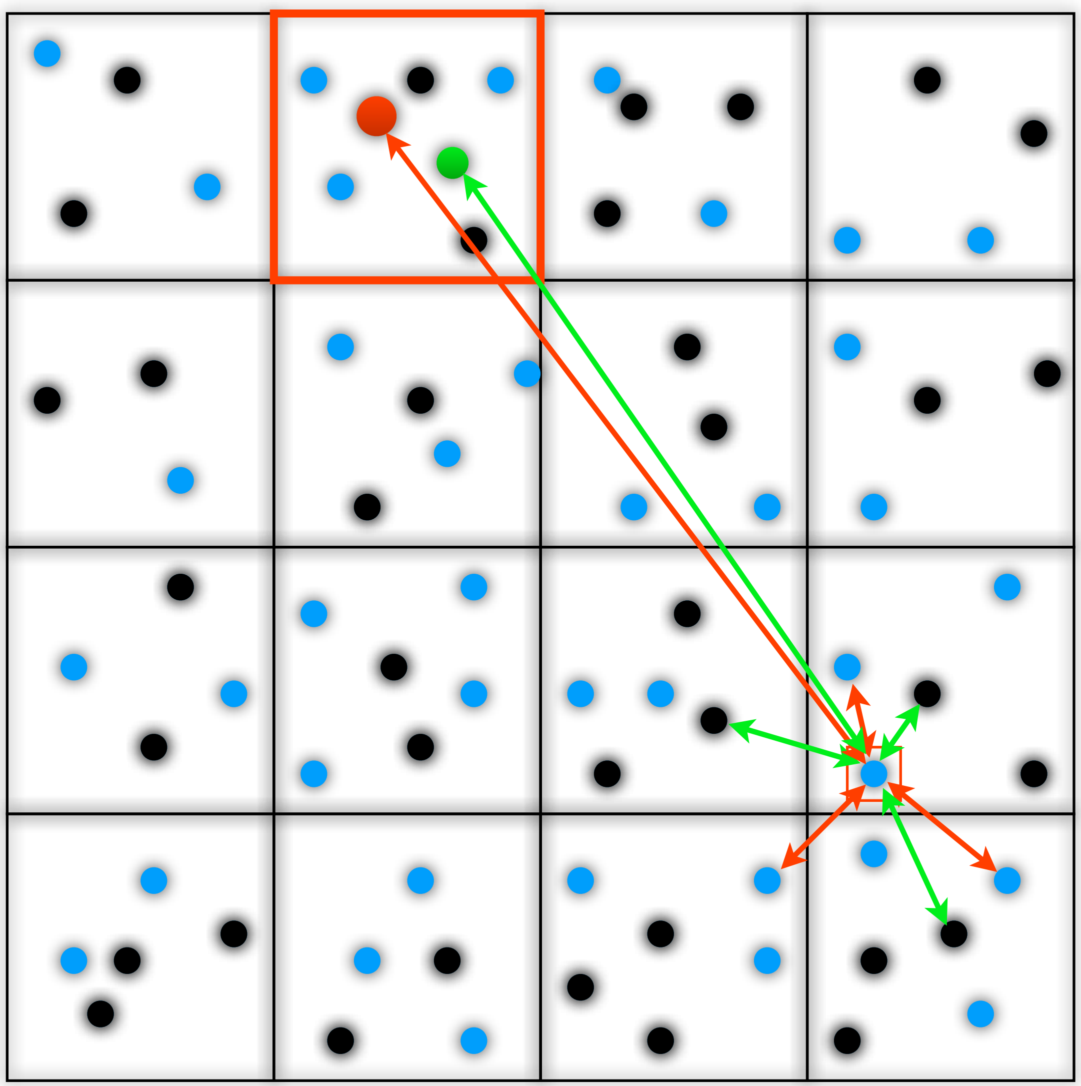

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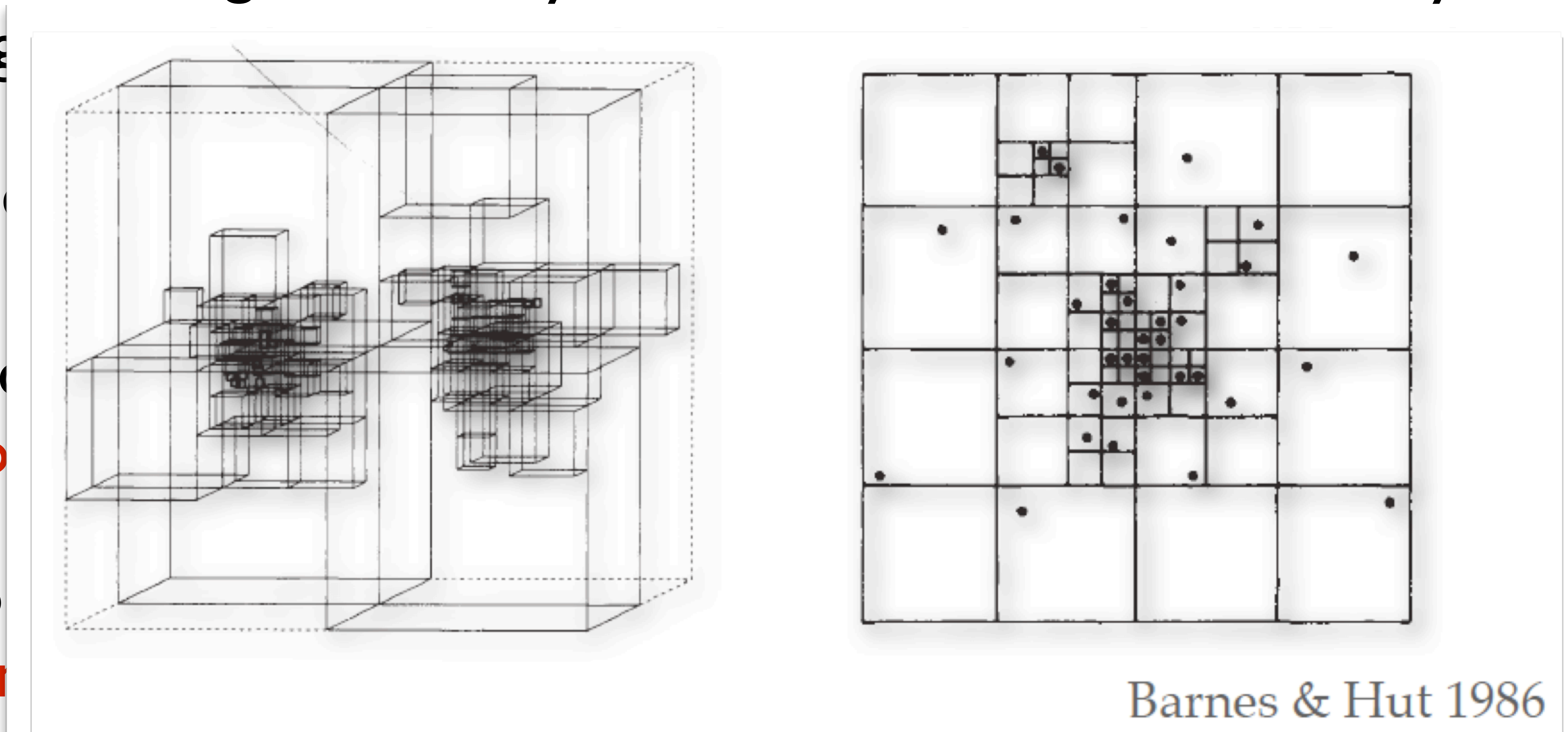

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- Employs multi-grid acceleration to achieve faster convergence

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• Once f_R is known, $\delta R(f_R)$ is also known, and the Poisson equation

$$\nabla^2\Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R$$

can be solved using the standard Gadget TreePM algorithm by:

i) associate an effective particle mass $m_{\delta R}$ to the density perturbations δR
ii) Apply the standard TreePM integration to the particles with mass $m + m_{\delta R}$

NON-LINEAR STRUCTURE FORMATION IN DARK ENERGY MODELS

Interacting Dark Energy

Non-universal couplings: Coupled Quintessence

$$\vec{a}_{\rm CDM} = -\vec{\nabla}\Phi_g(1 + 2\beta^2(\phi)) + \beta(\phi)\dot{\phi}\vec{v} \qquad \vec{a}_{\rm b} = -\vec{\nabla}\Phi_g$$

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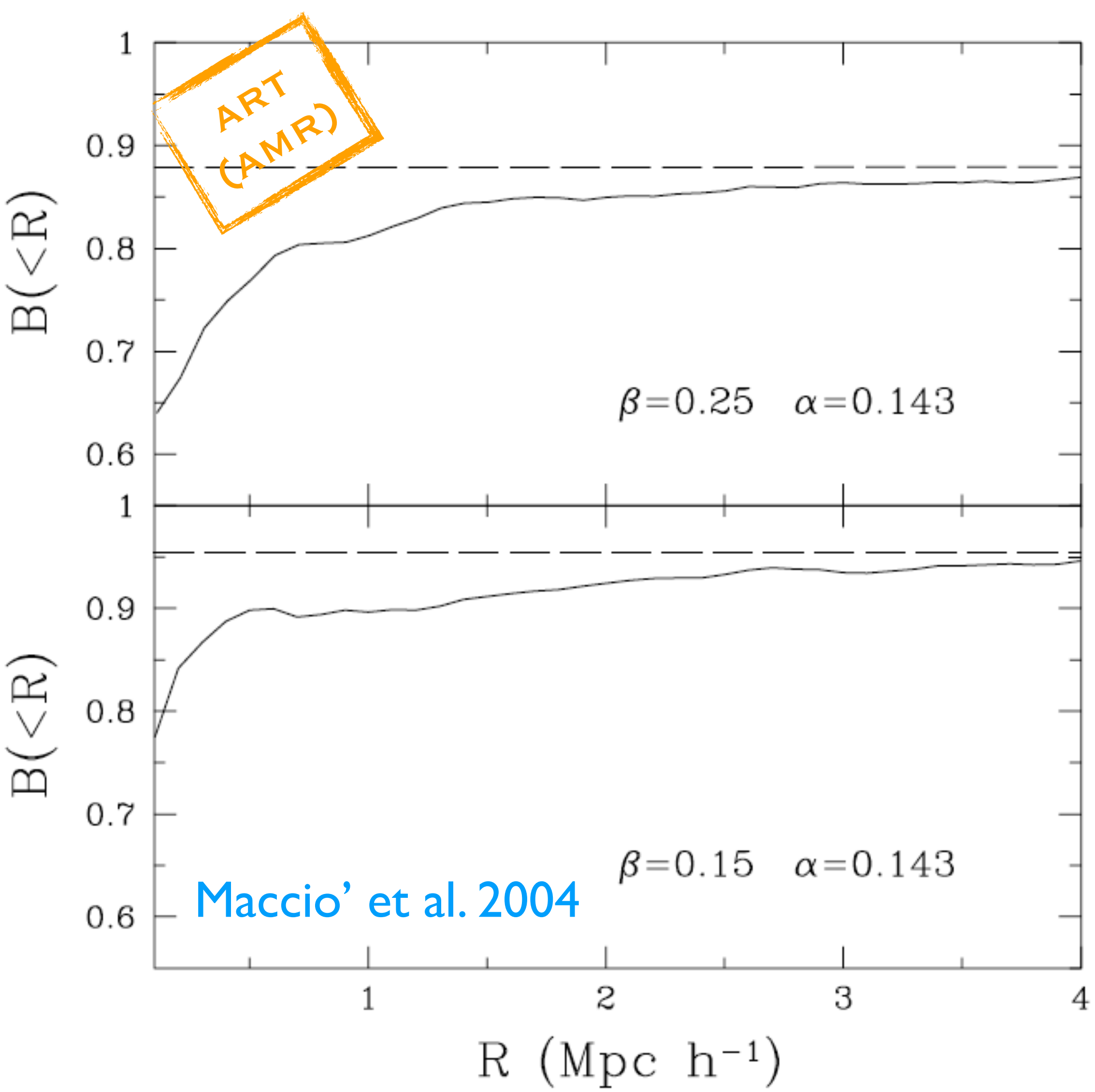
$$\vec{a}_{\rm CDM} = -\vec{\nabla}\Phi_g\left(1 + 2\beta^2(\phi)\right) + \beta(\phi)\dot{\phi}\vec{v} \qquad \vec{a}_{\rm b} = -\vec{\nabla}\Phi_g$$

First N-body simulations by Macciò et al. 2004 using ART



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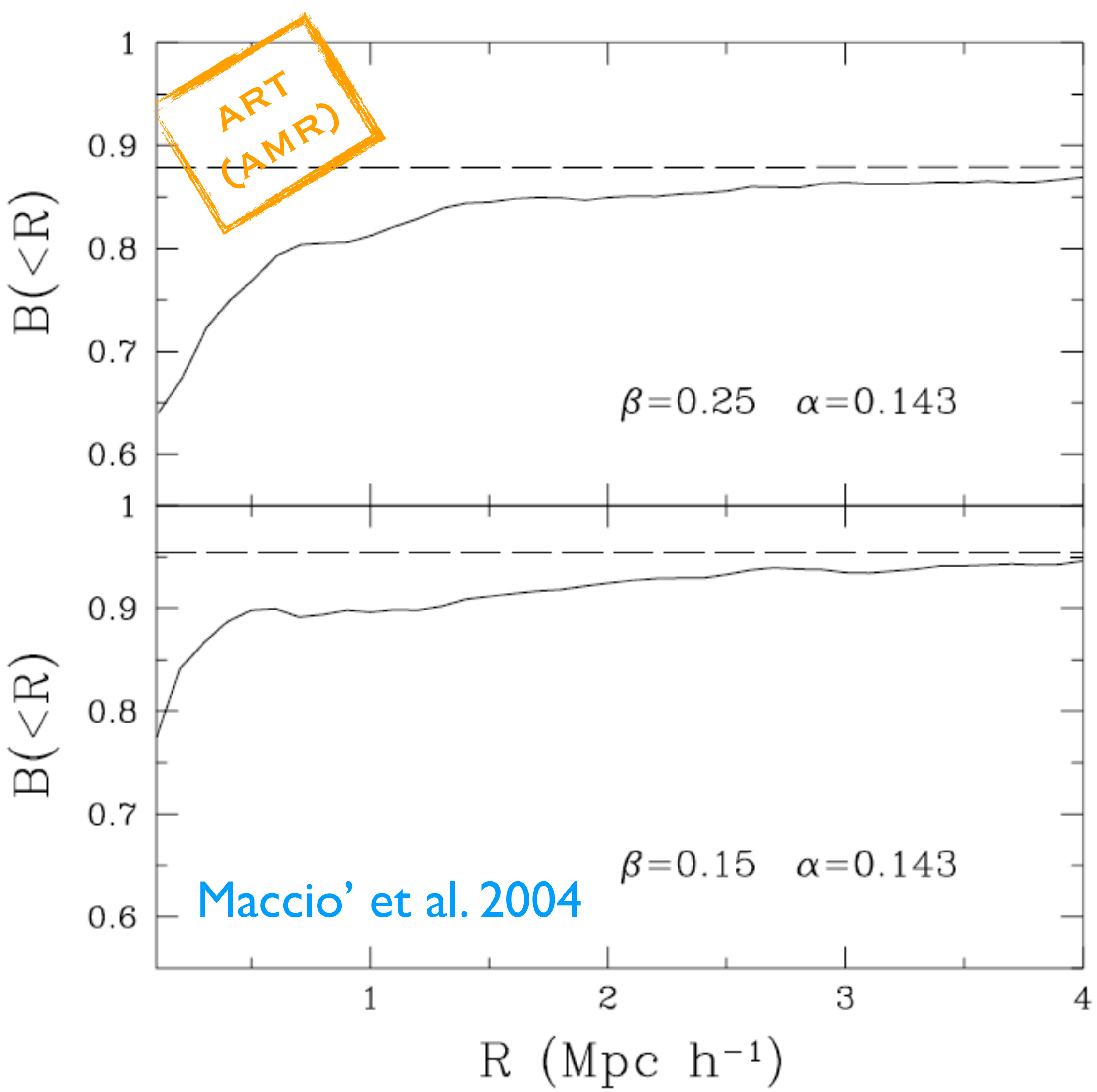

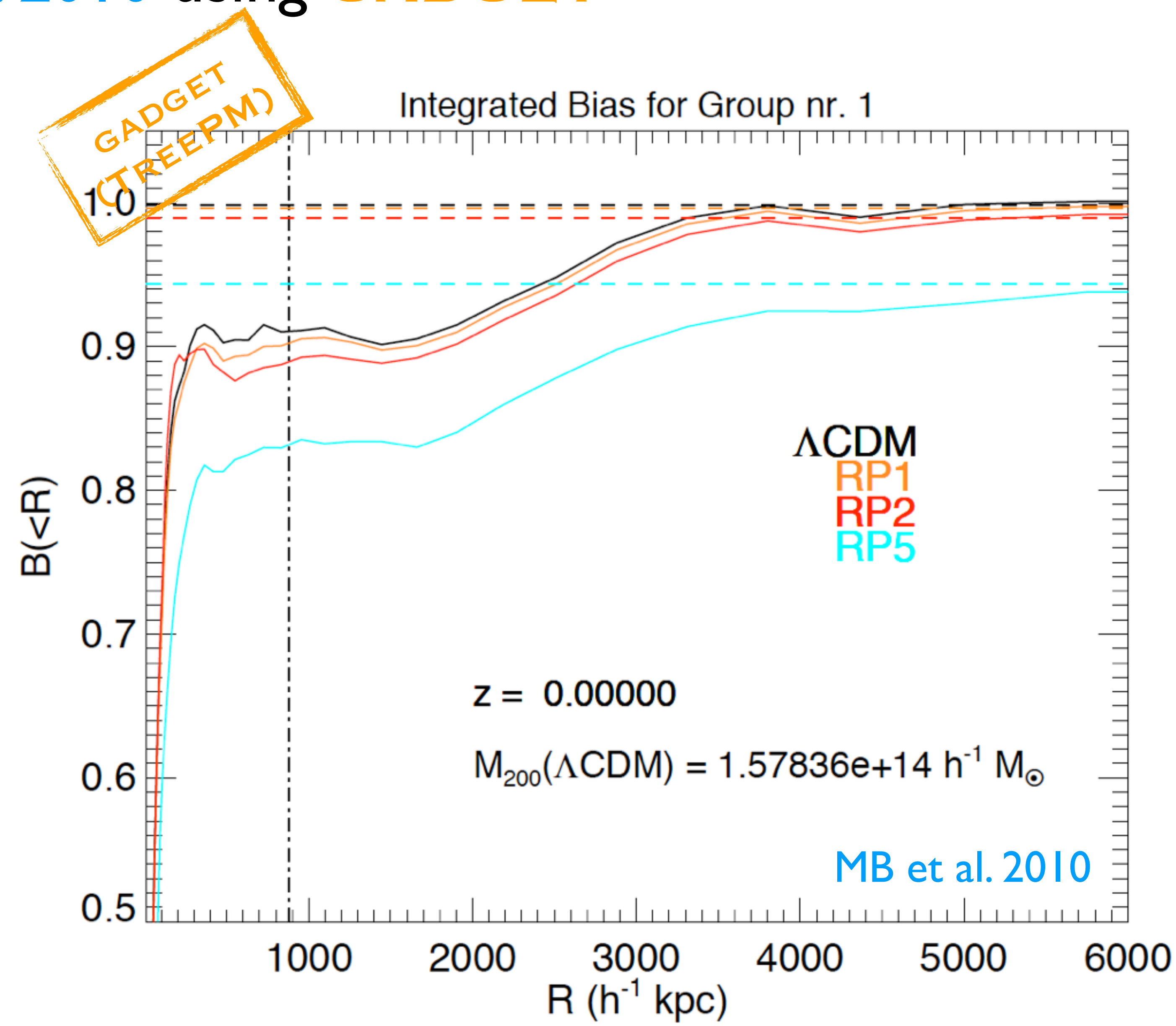

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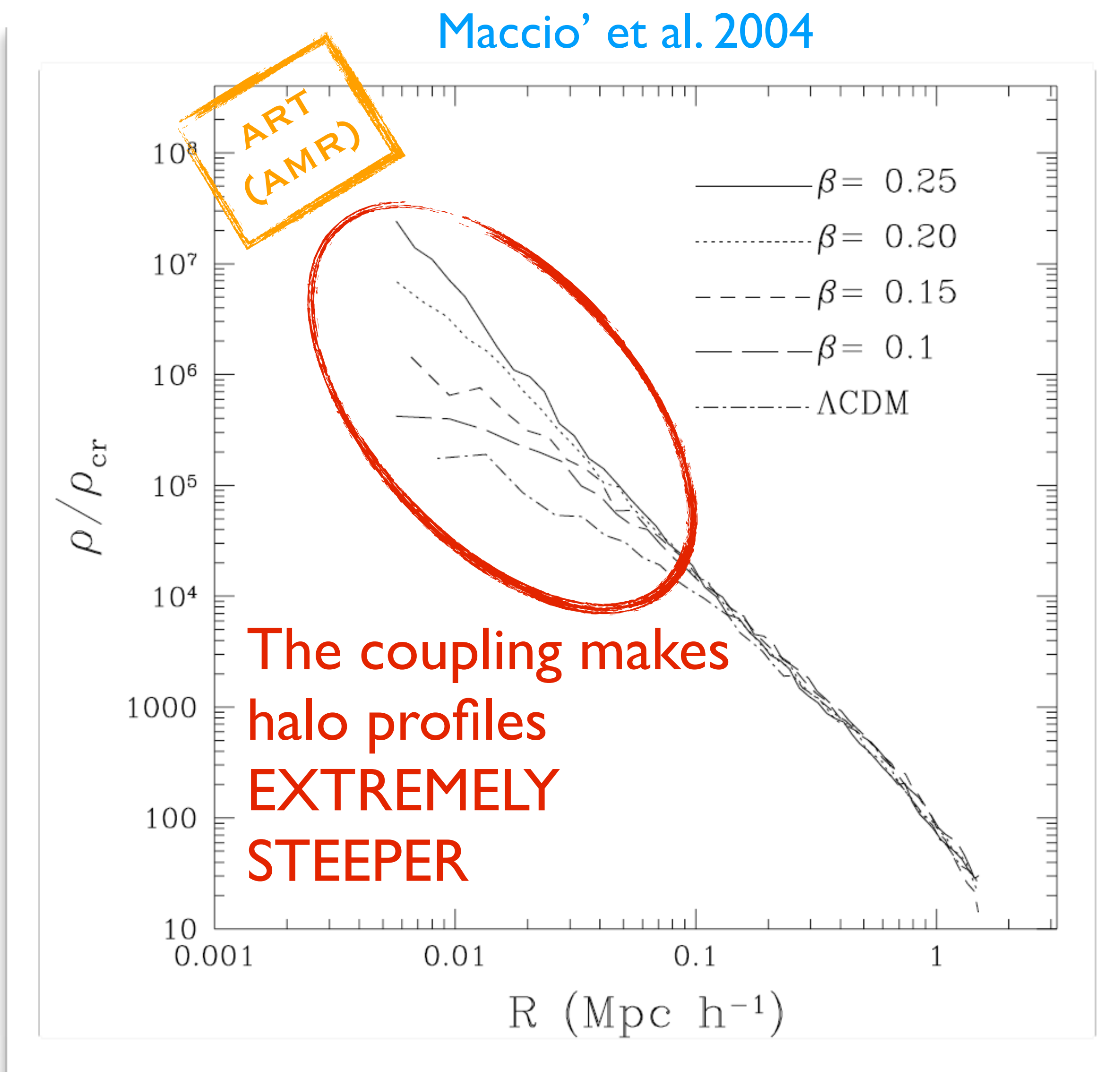

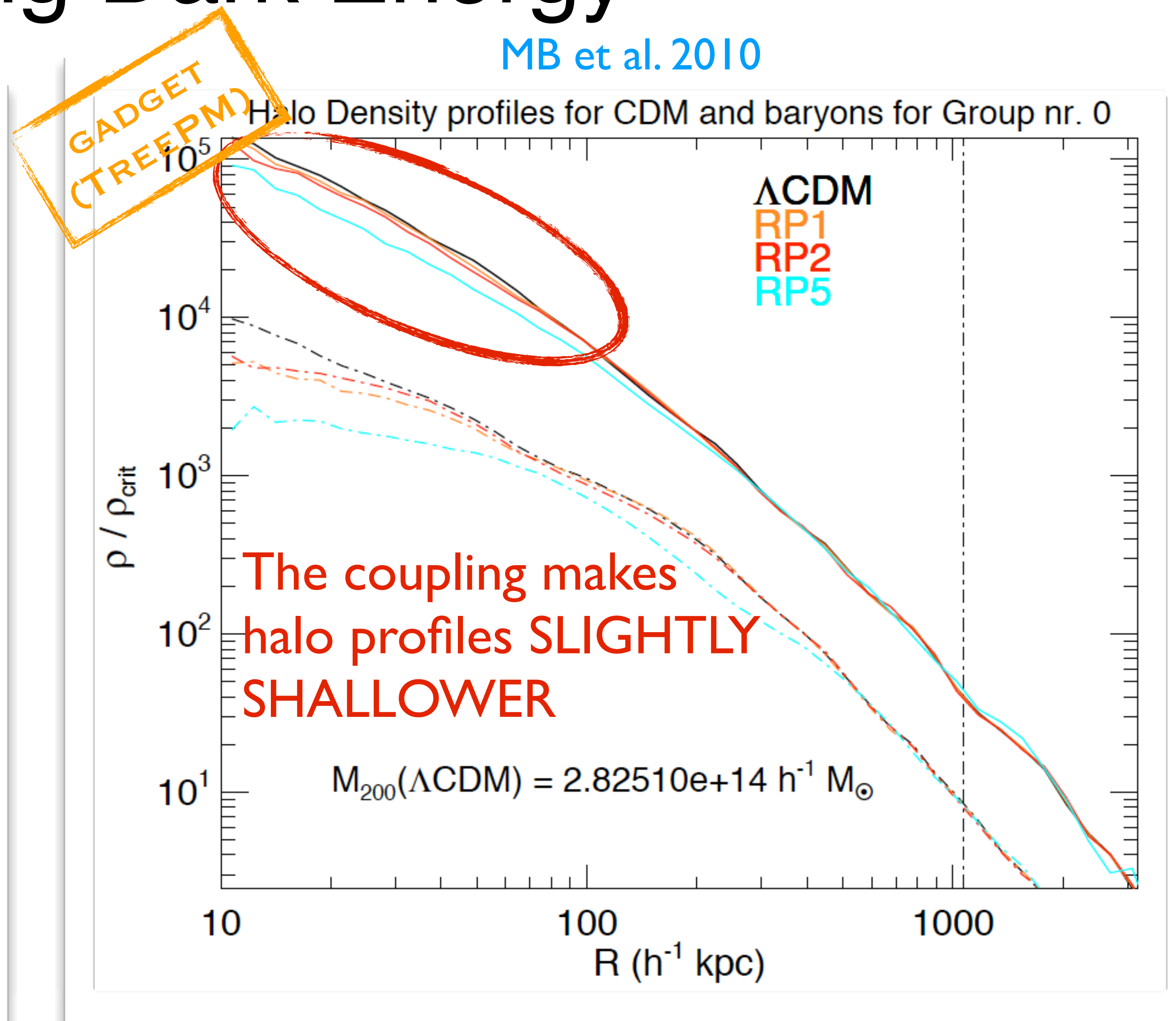

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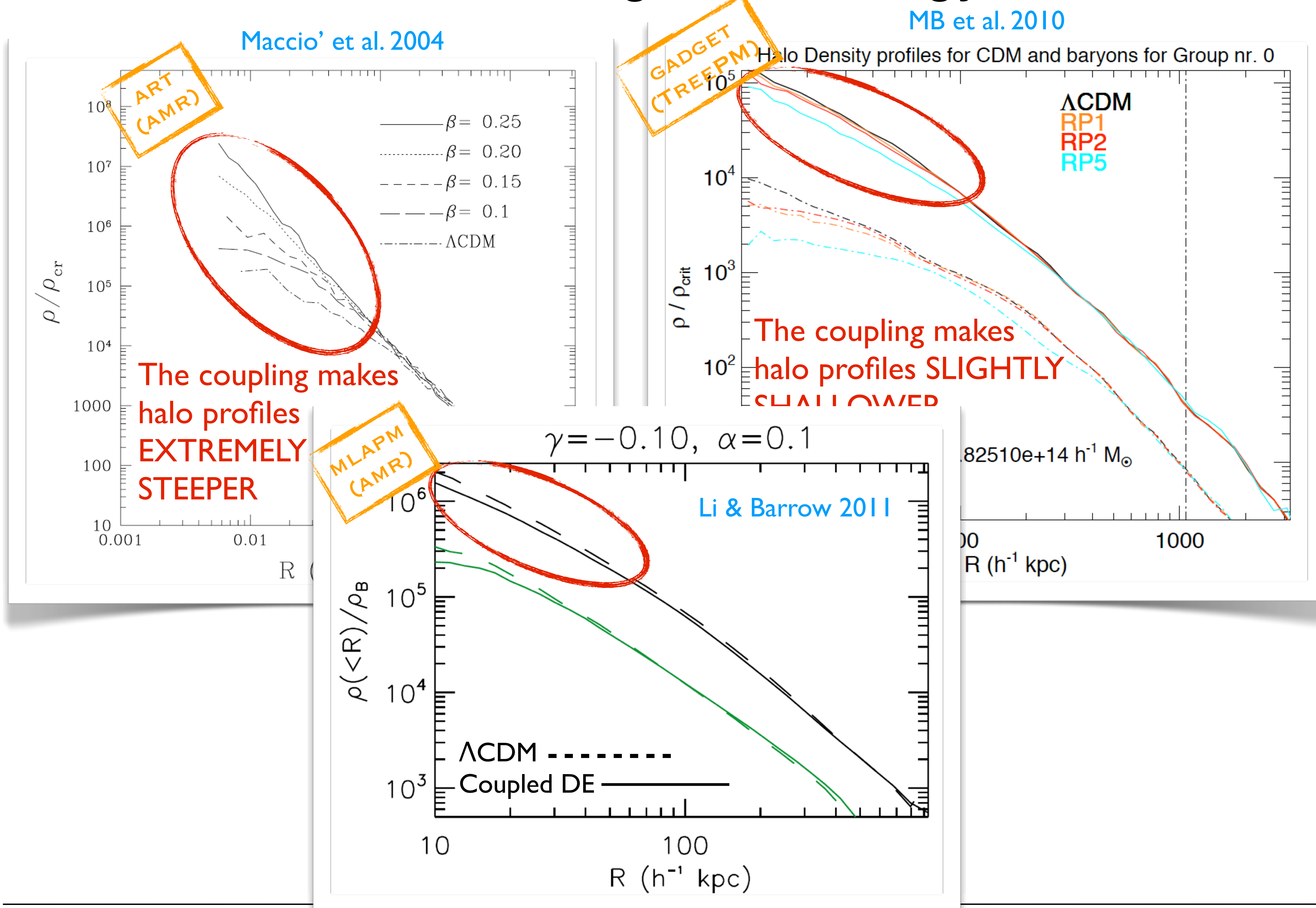

Interacting Dark Energy

The abundance of high-z massive clusters: MB 2012, arXiv:1107.5049

Are high-z massive clusters in tension with ΛCDM?

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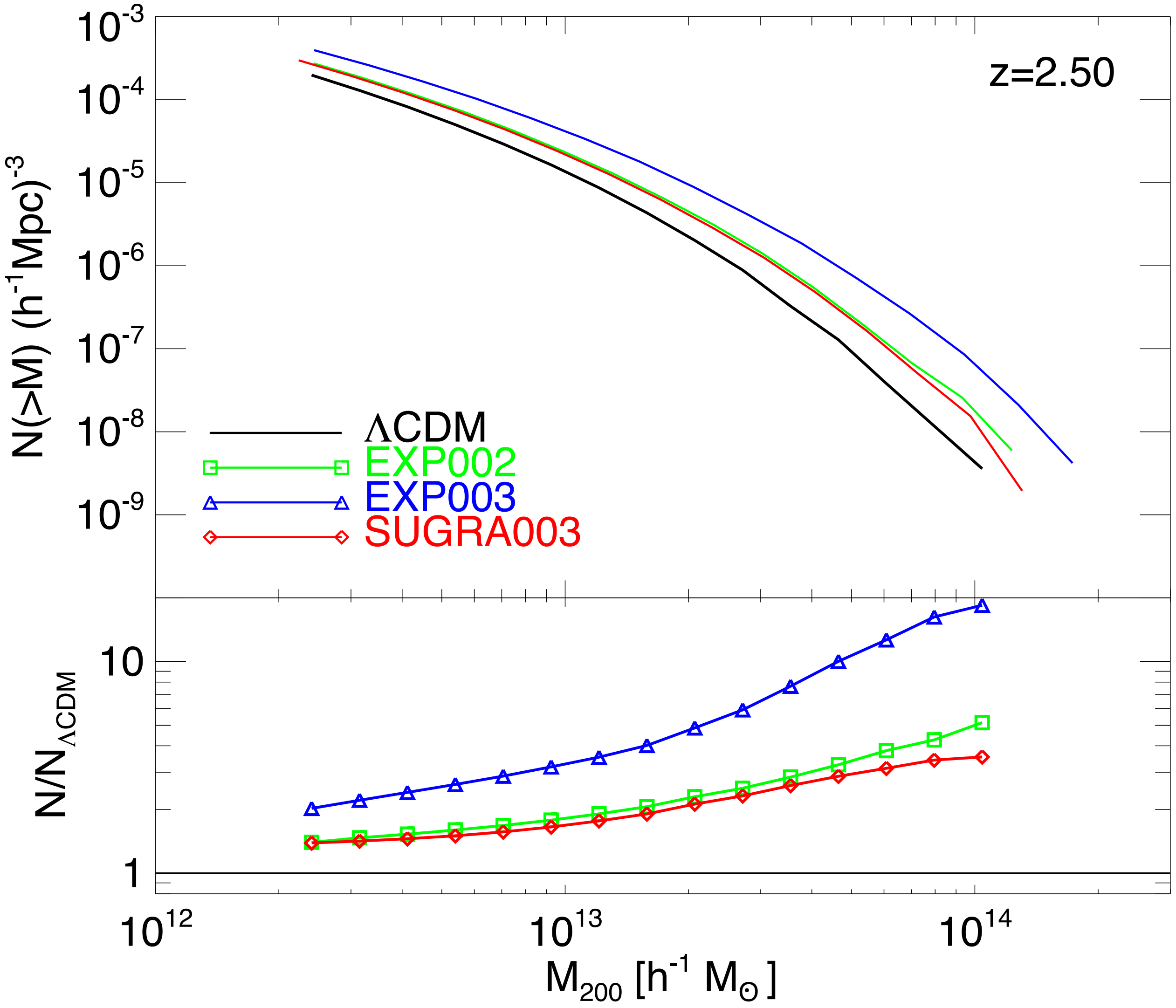

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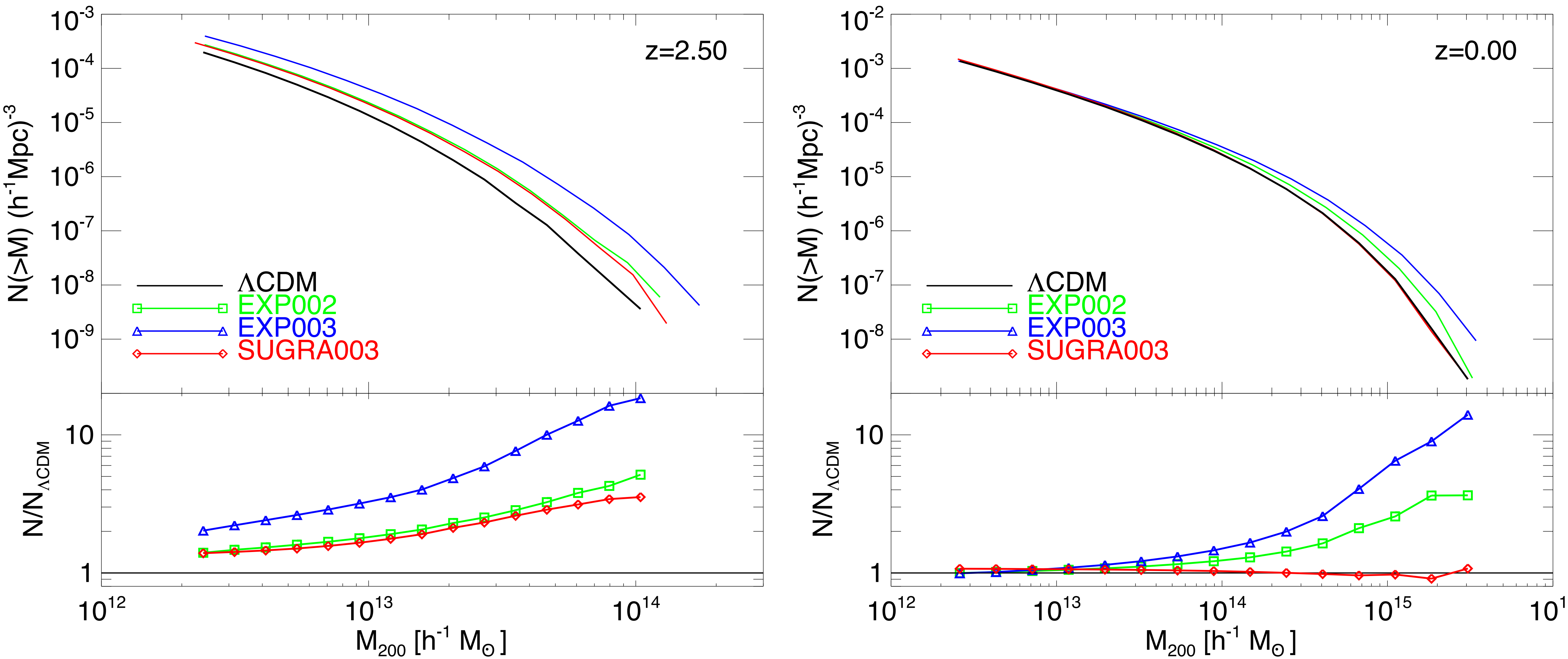

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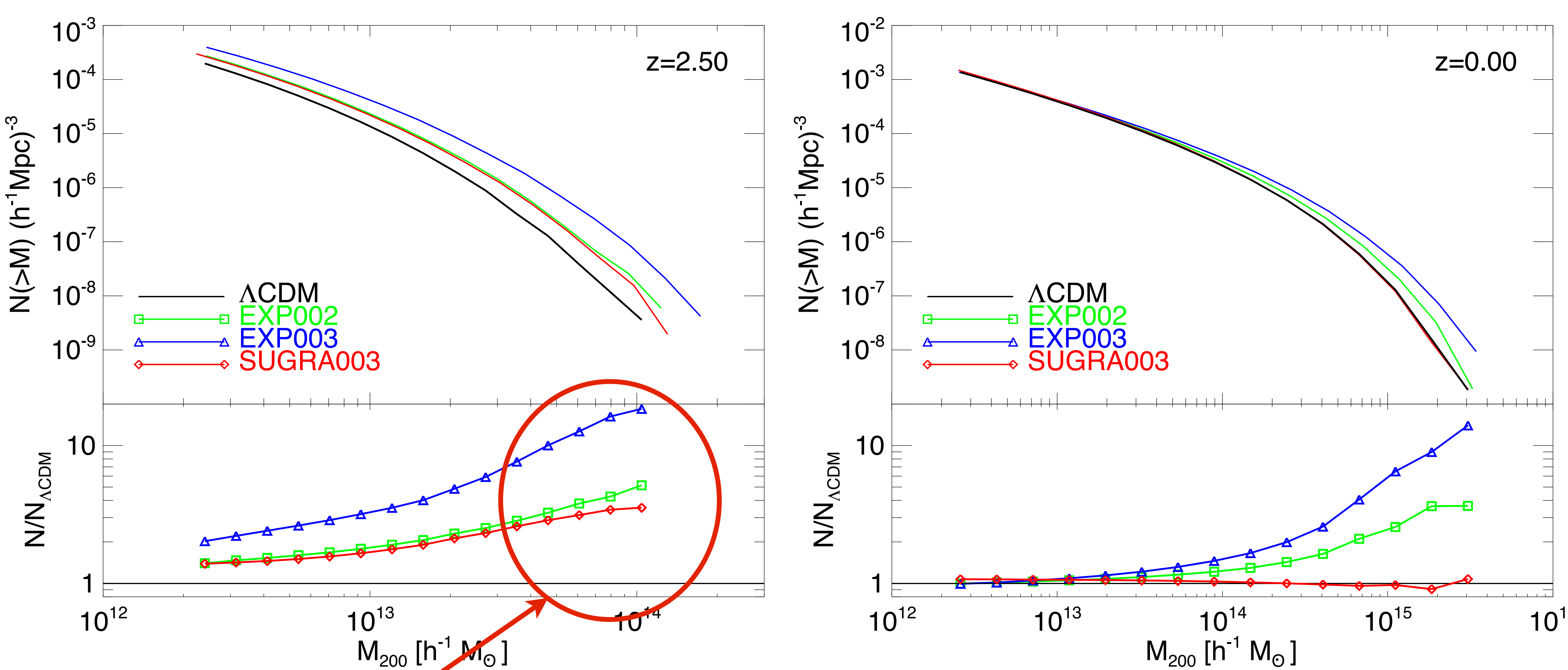

More massive clusters at high z

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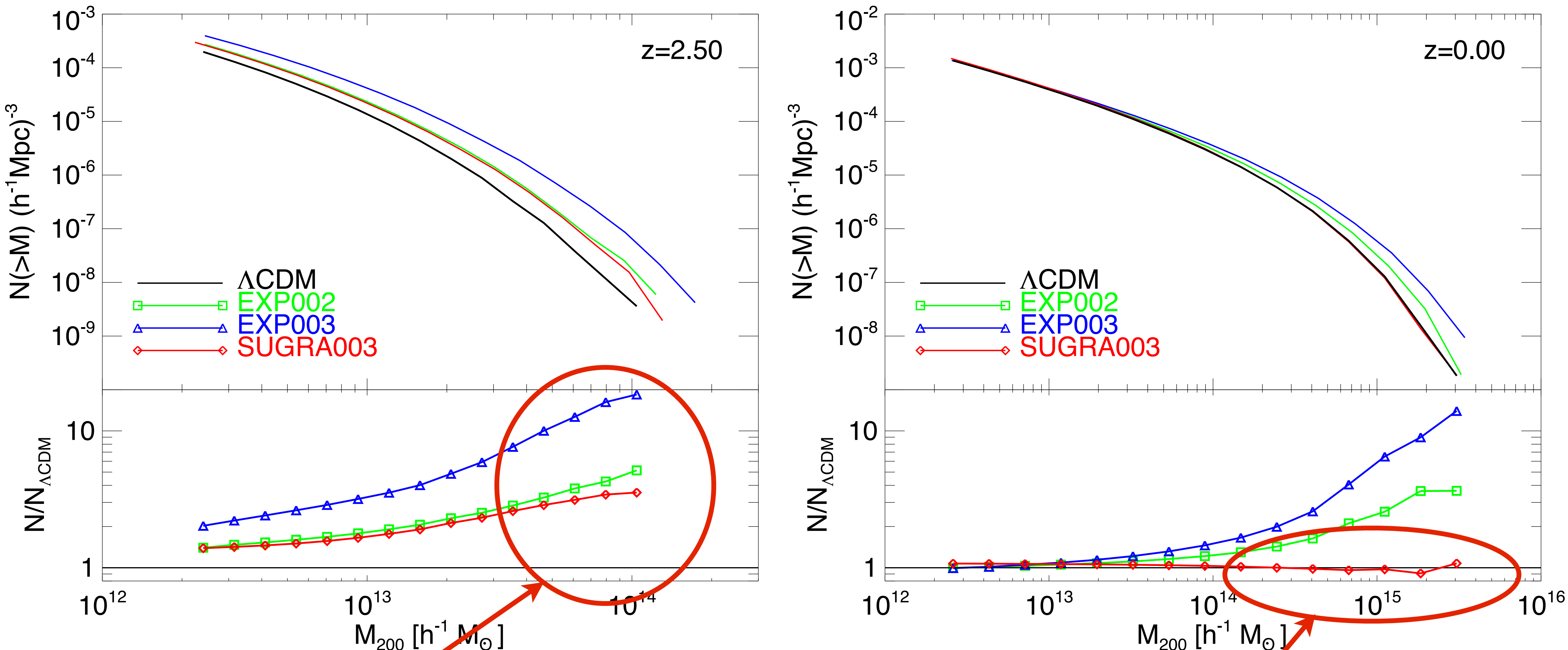


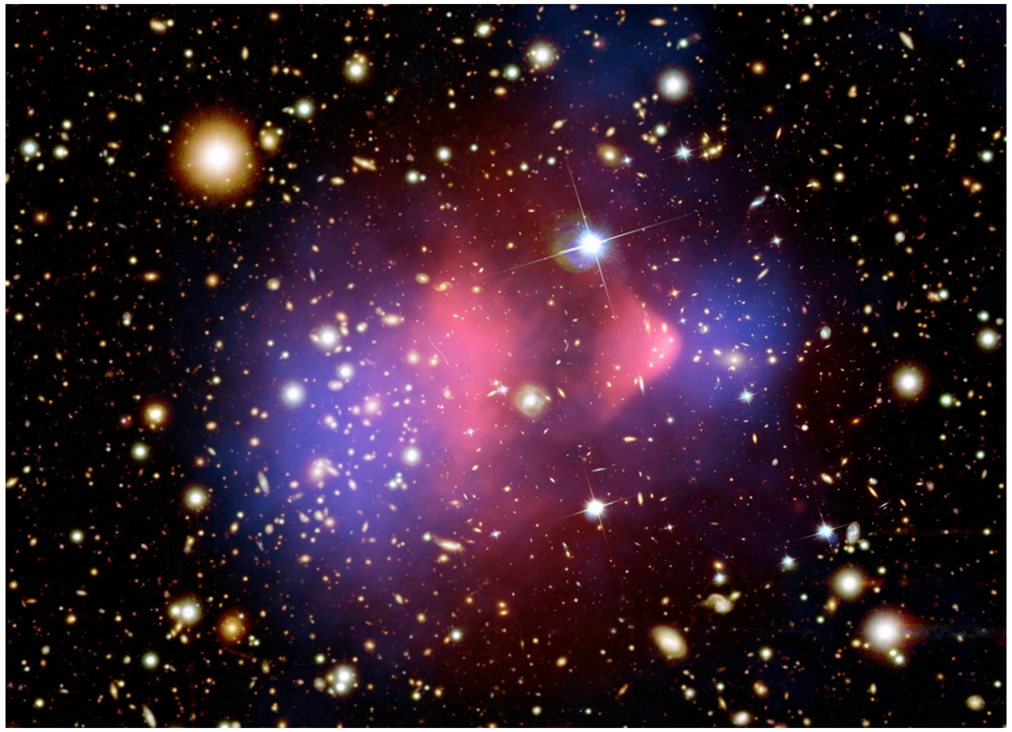
More massive clusters at high z

Same number of clusters as ΛCDM at z=0

Interacting Dark Energy

Infall velocity of colliding bullet-like clusters, Lee & MB 2012, arXiv:1110.0015

Is the bullet cluster in tension with ΛCDM?



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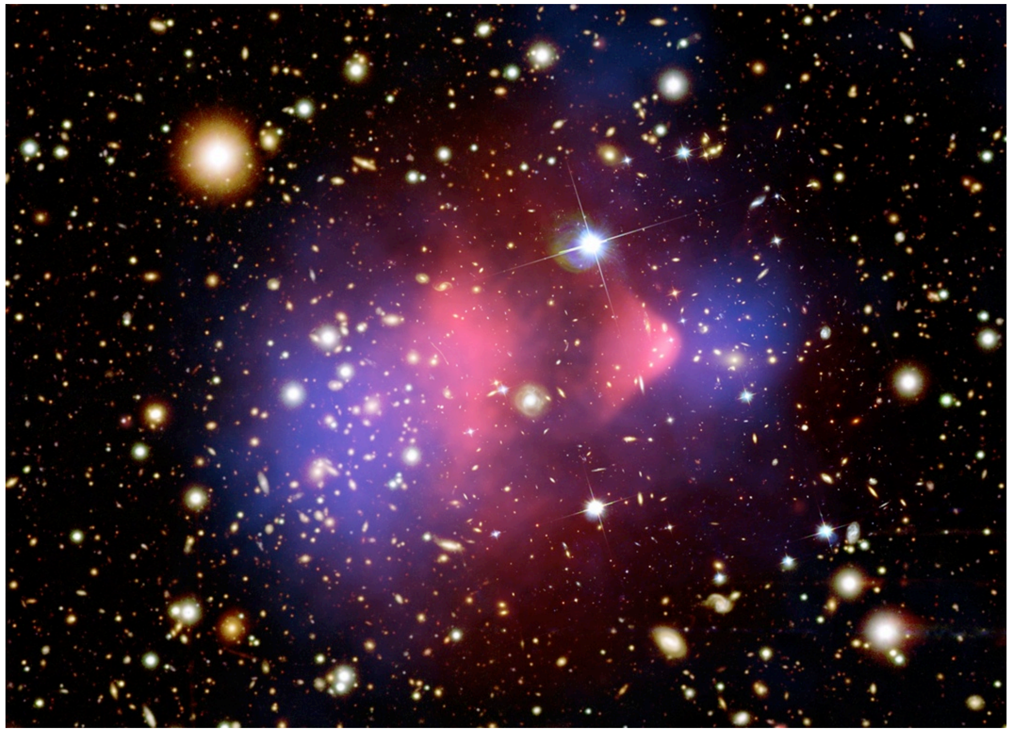

Colliding galaxy clusters with comparable mass, high infall velocity, and low impact parameter are expected to be very rare in ΛCDM (Lee & Komatsu 2010)

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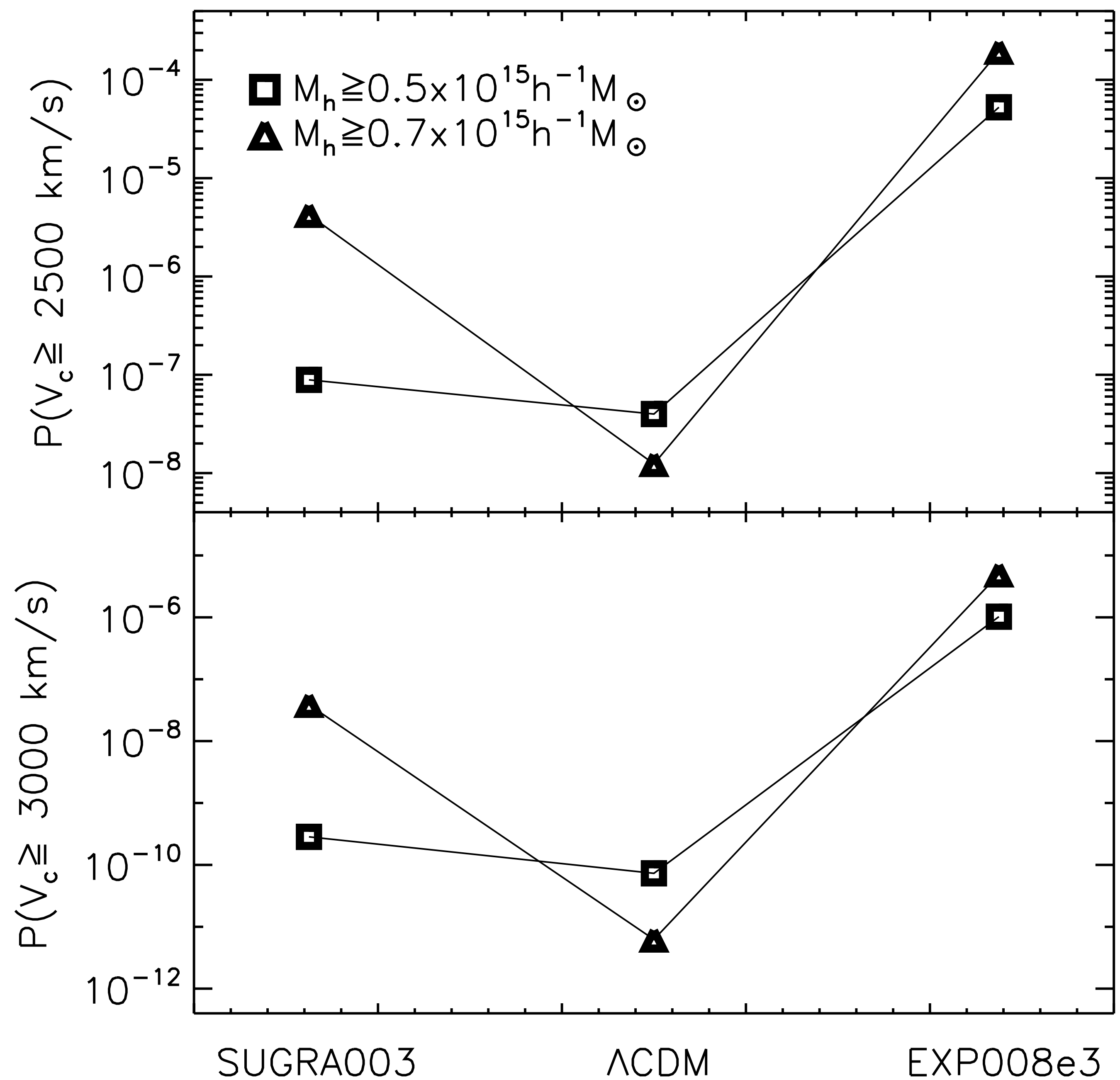
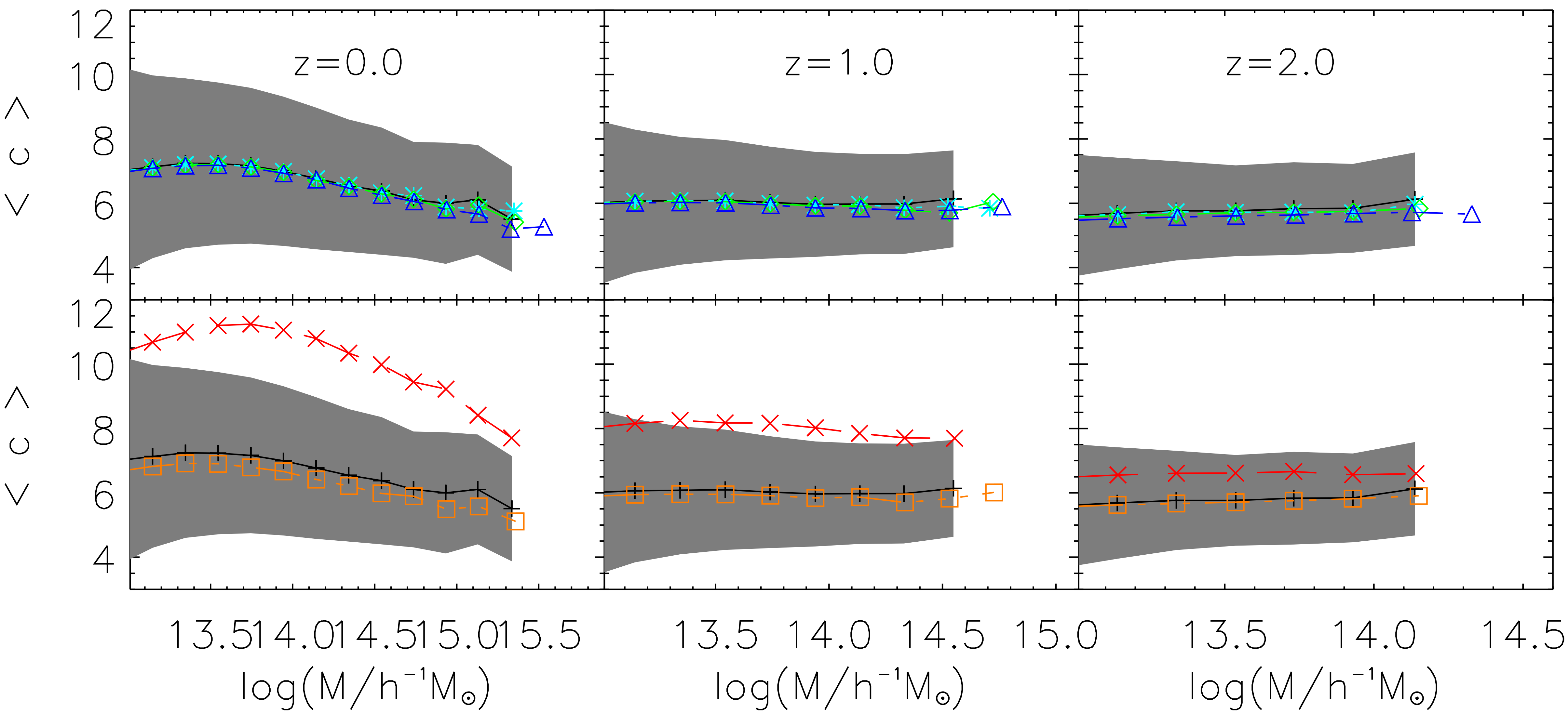


Interacting Dark Energy

The concentration-mass relation from **CoDECS** (Cui et al. 2012):



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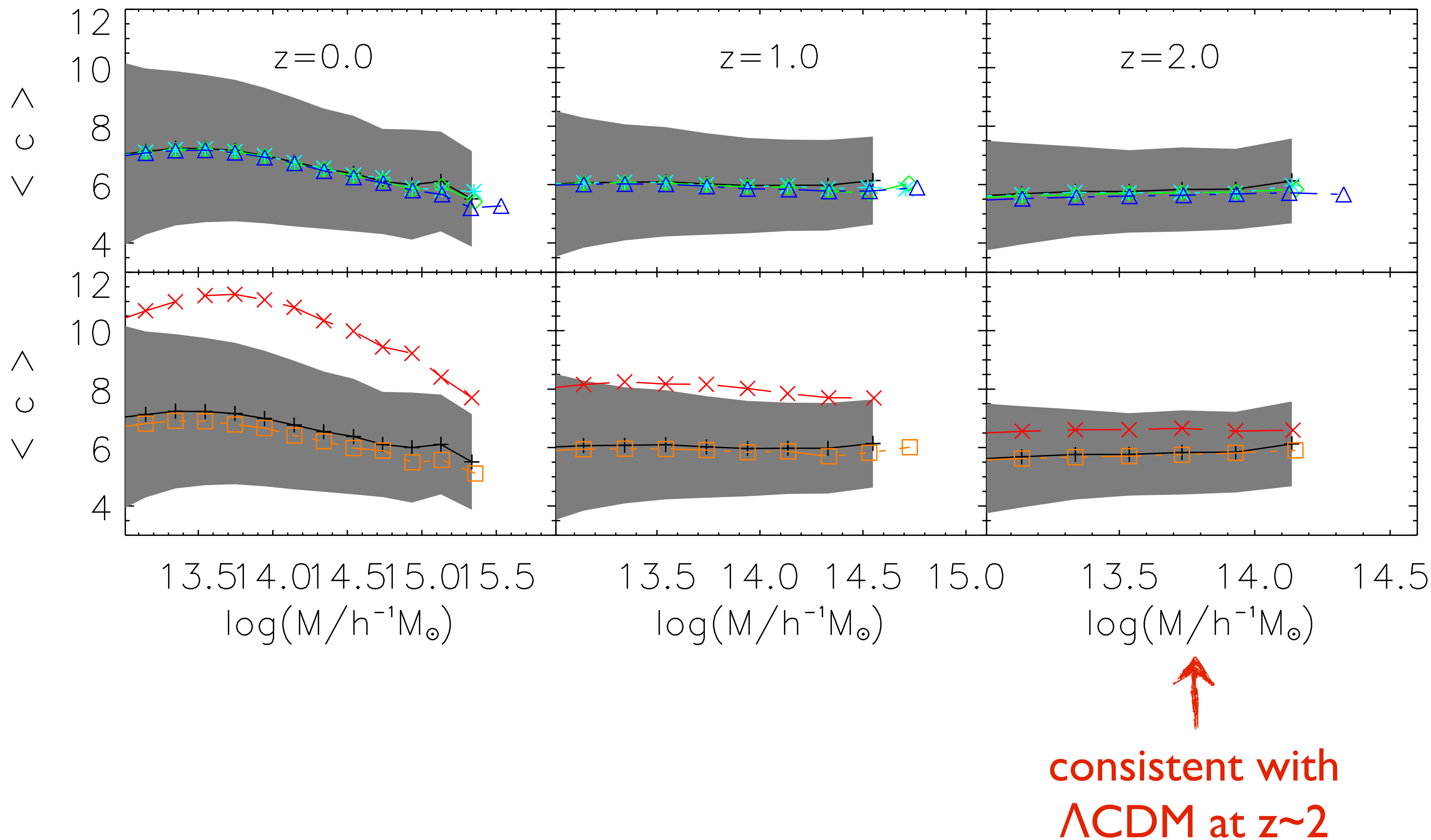

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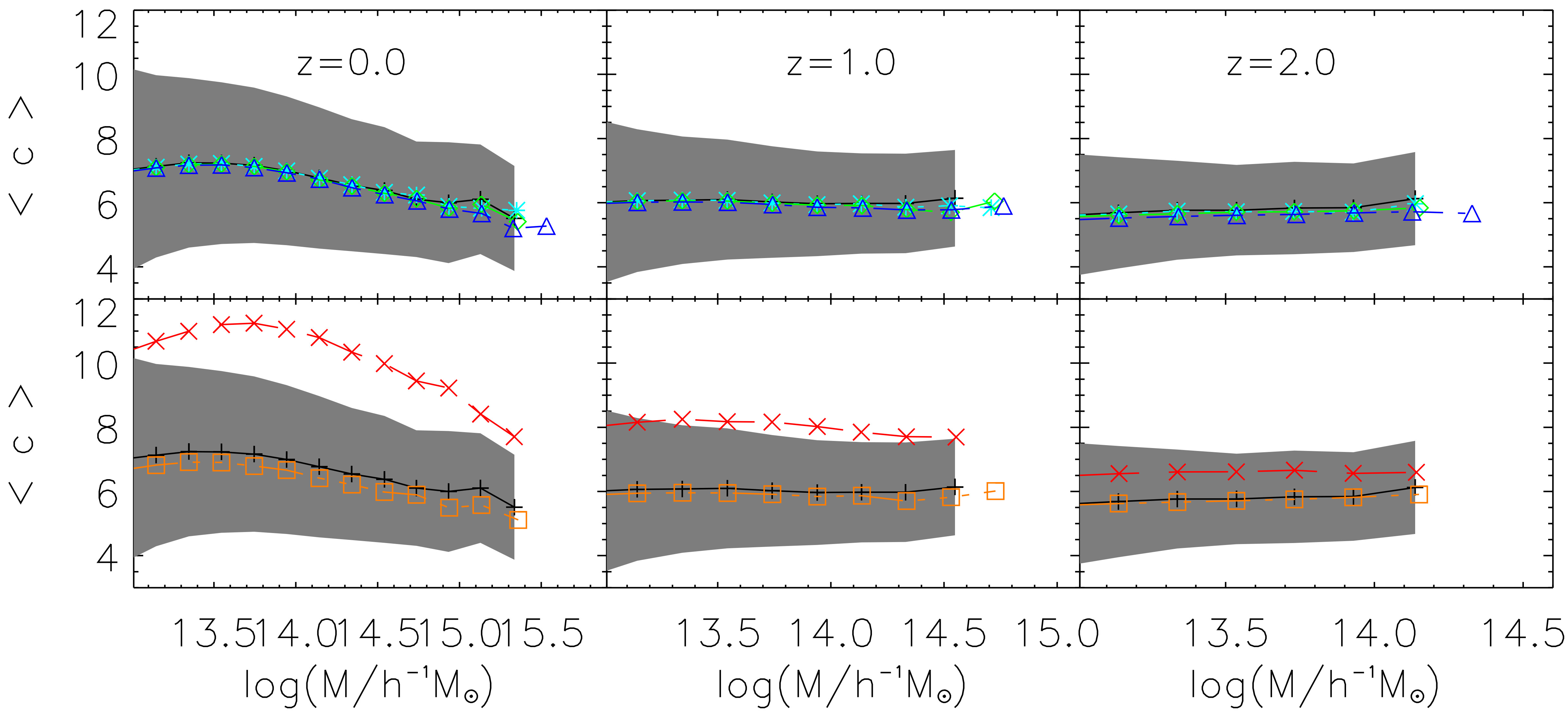

1σ higher than ΛCDM at z~1

consistent with ΛCDM at z~2

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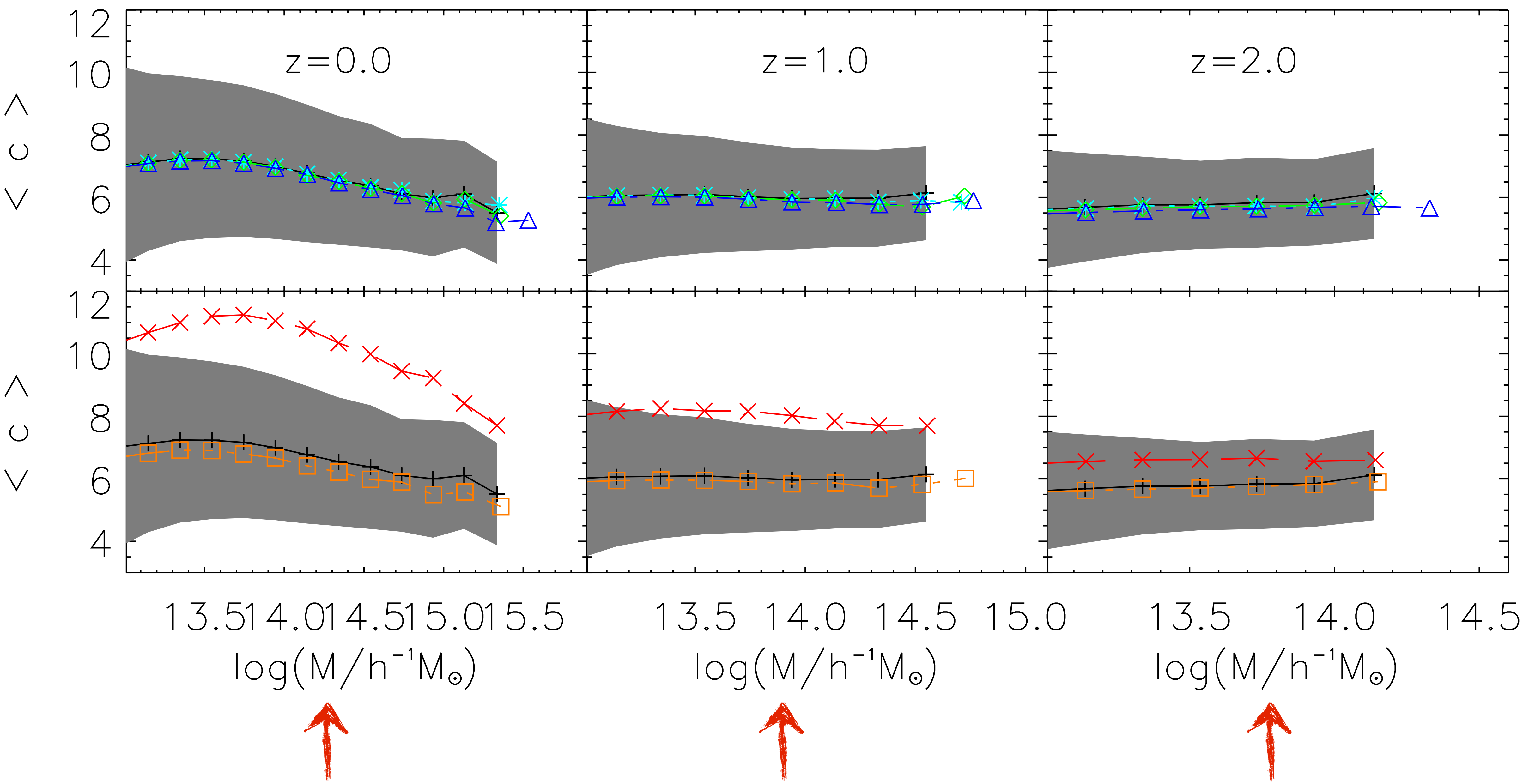


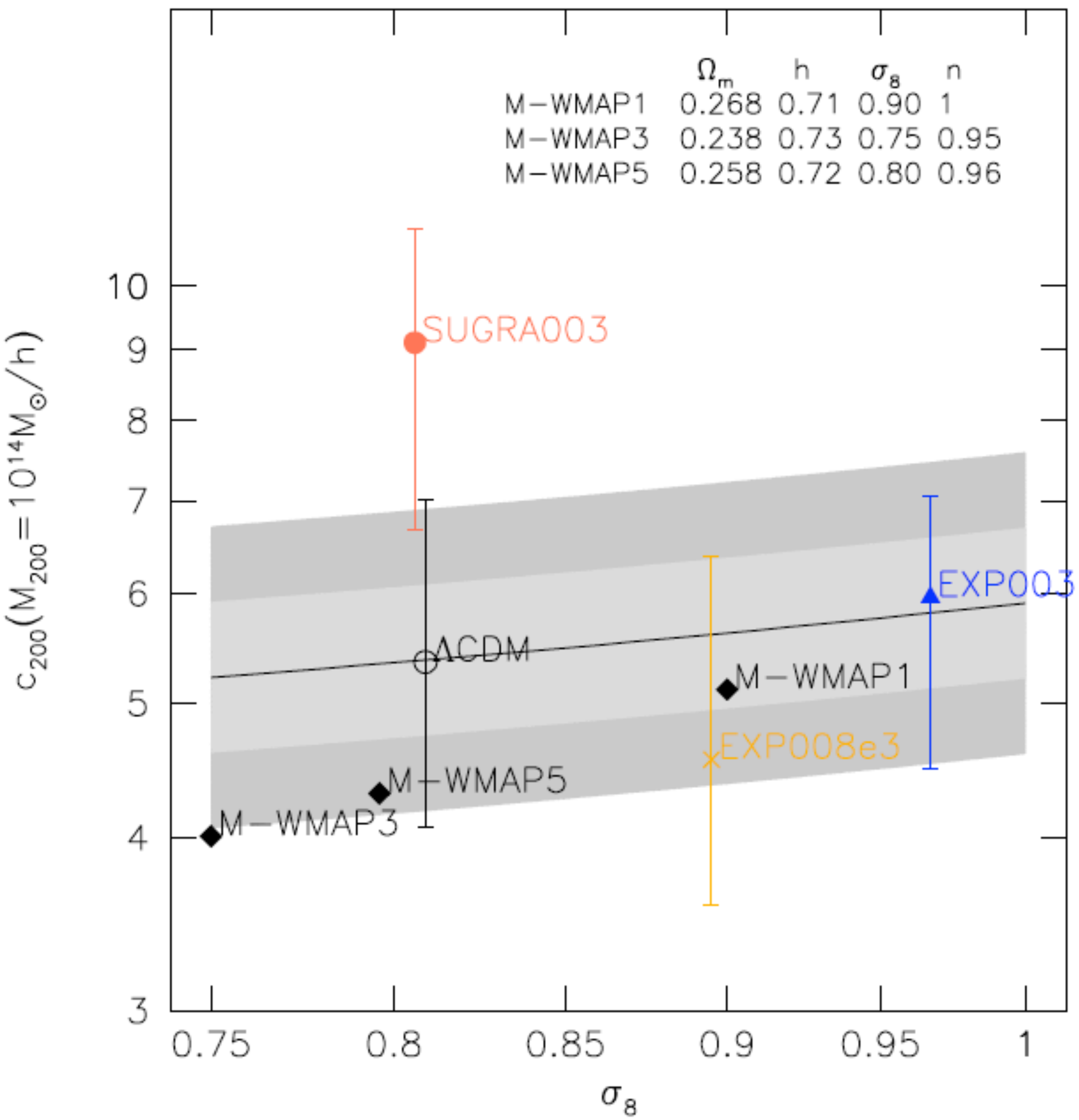
significantly higher than ΛCDM at z~0

1σ higher than ΛCDM at z~1

consistent with ΛCDM at z~2

Interacting Dark Energy

Breaking the c-σ_8 degeneracy for some specific cDE realizations



From Giocoli et al. 2013 (arXiv:1301.3151)

Interacting Dark Energy

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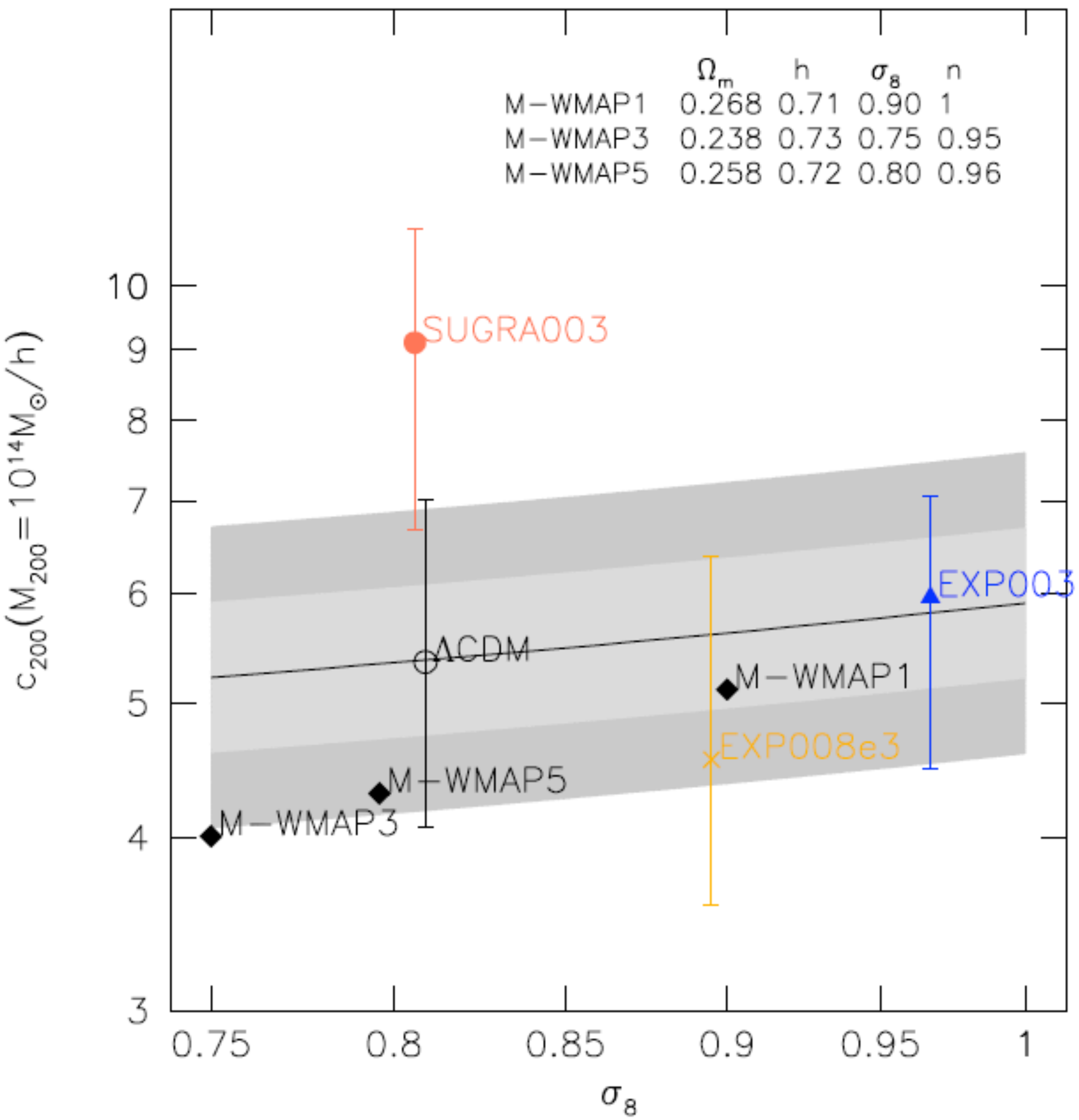
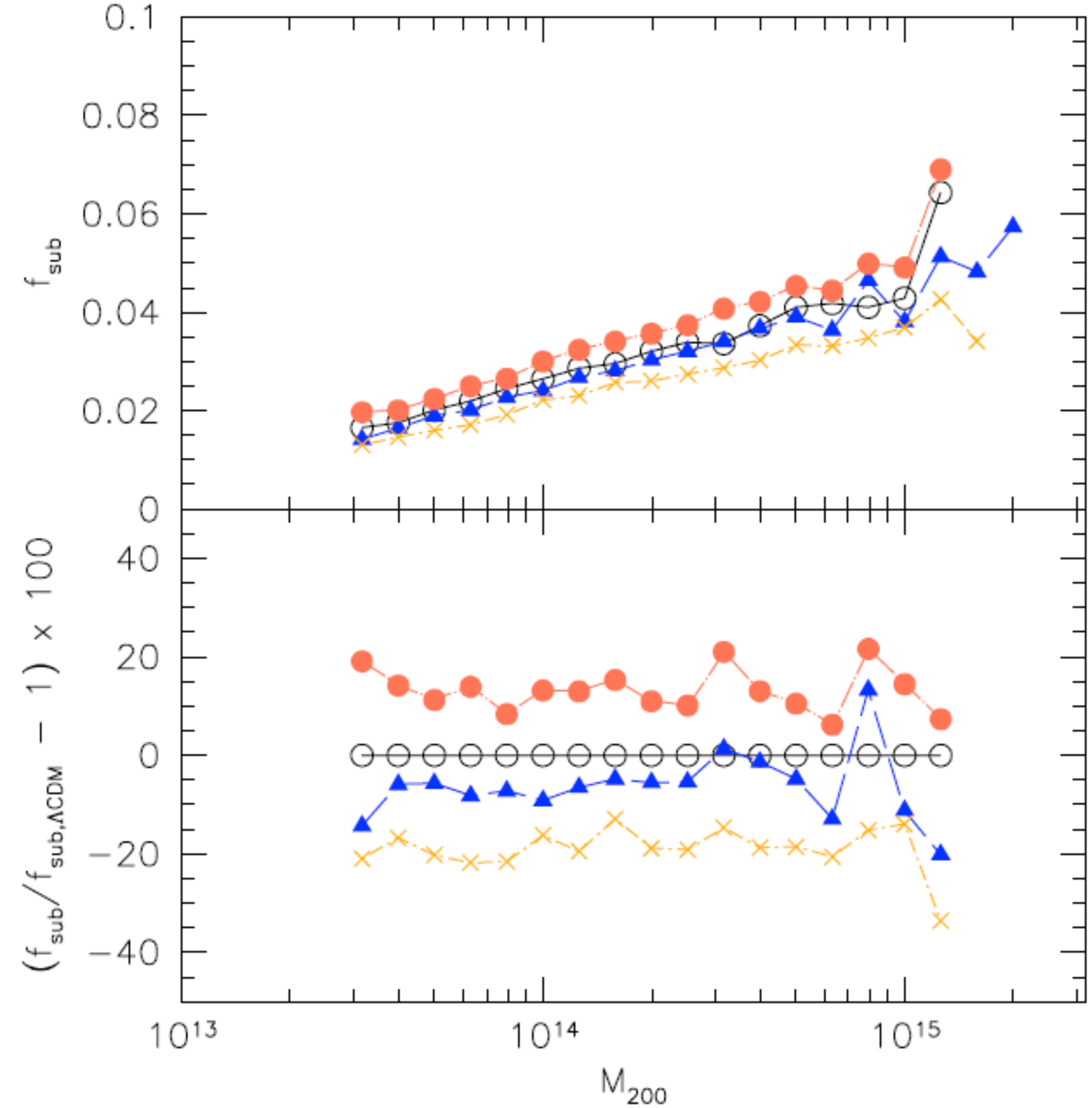


Substructure abundance discriminates among different cDE models



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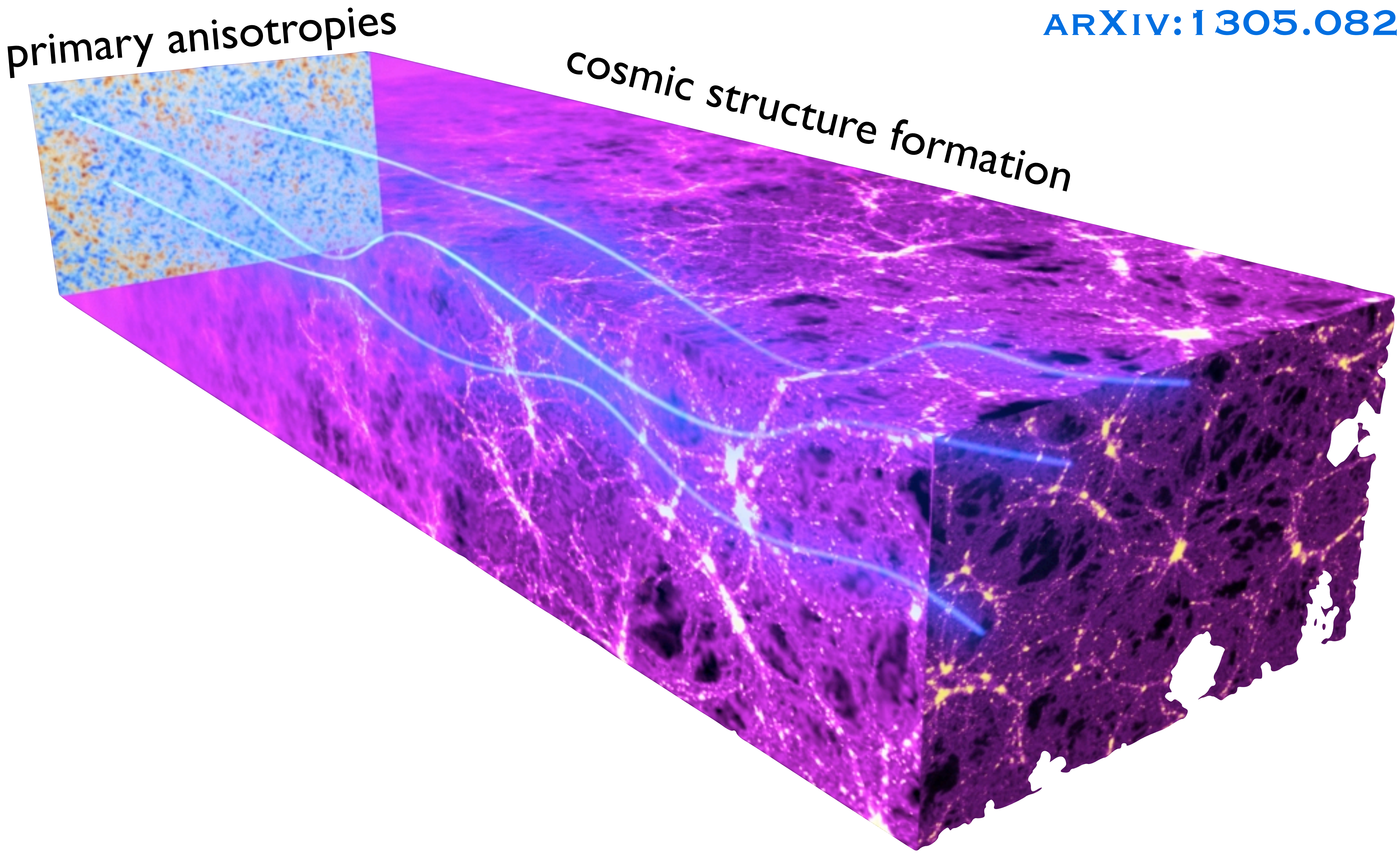
Interacting Dark Energy

CARBONE ET AL.,2013, ARXIV:1305.0829



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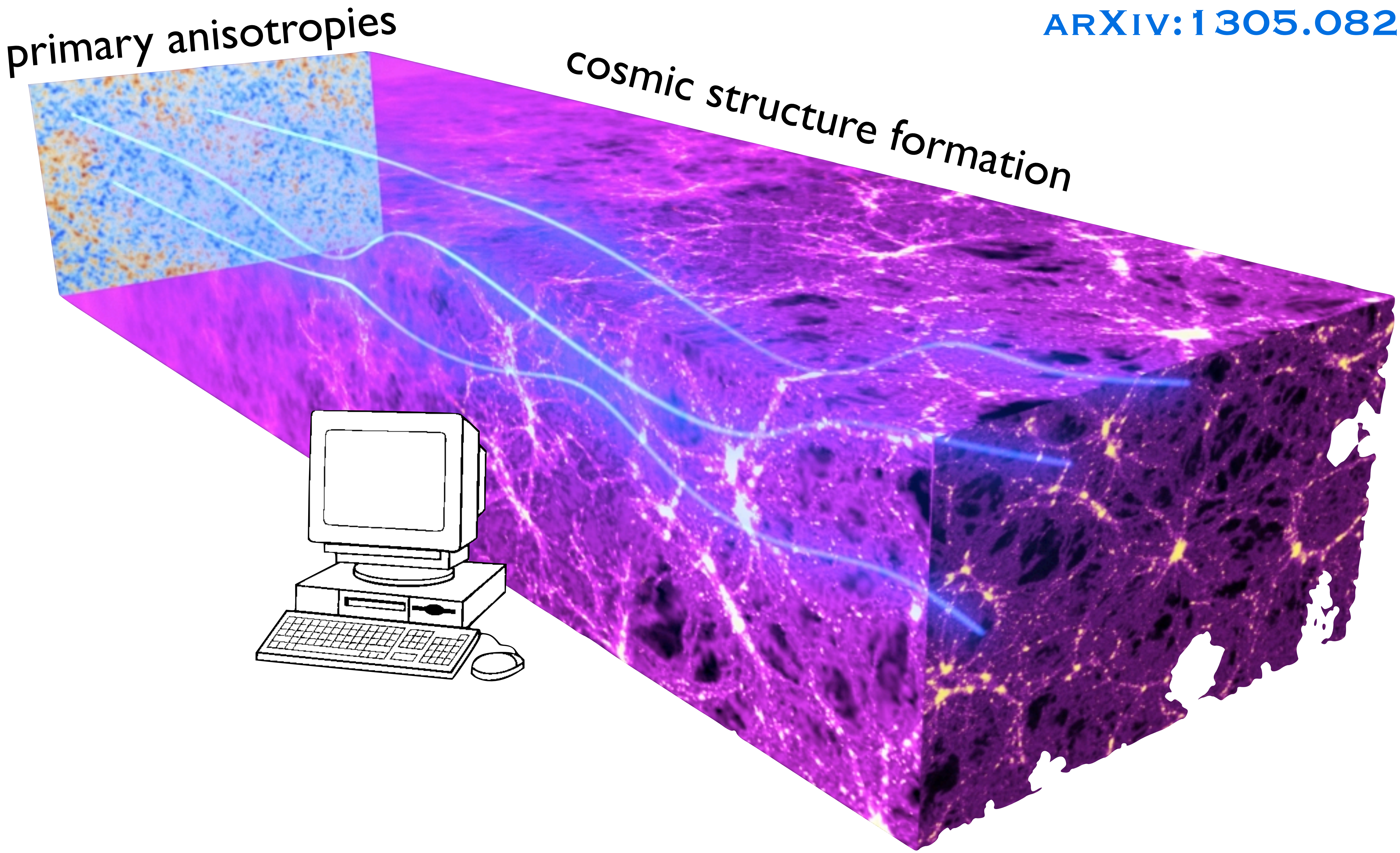

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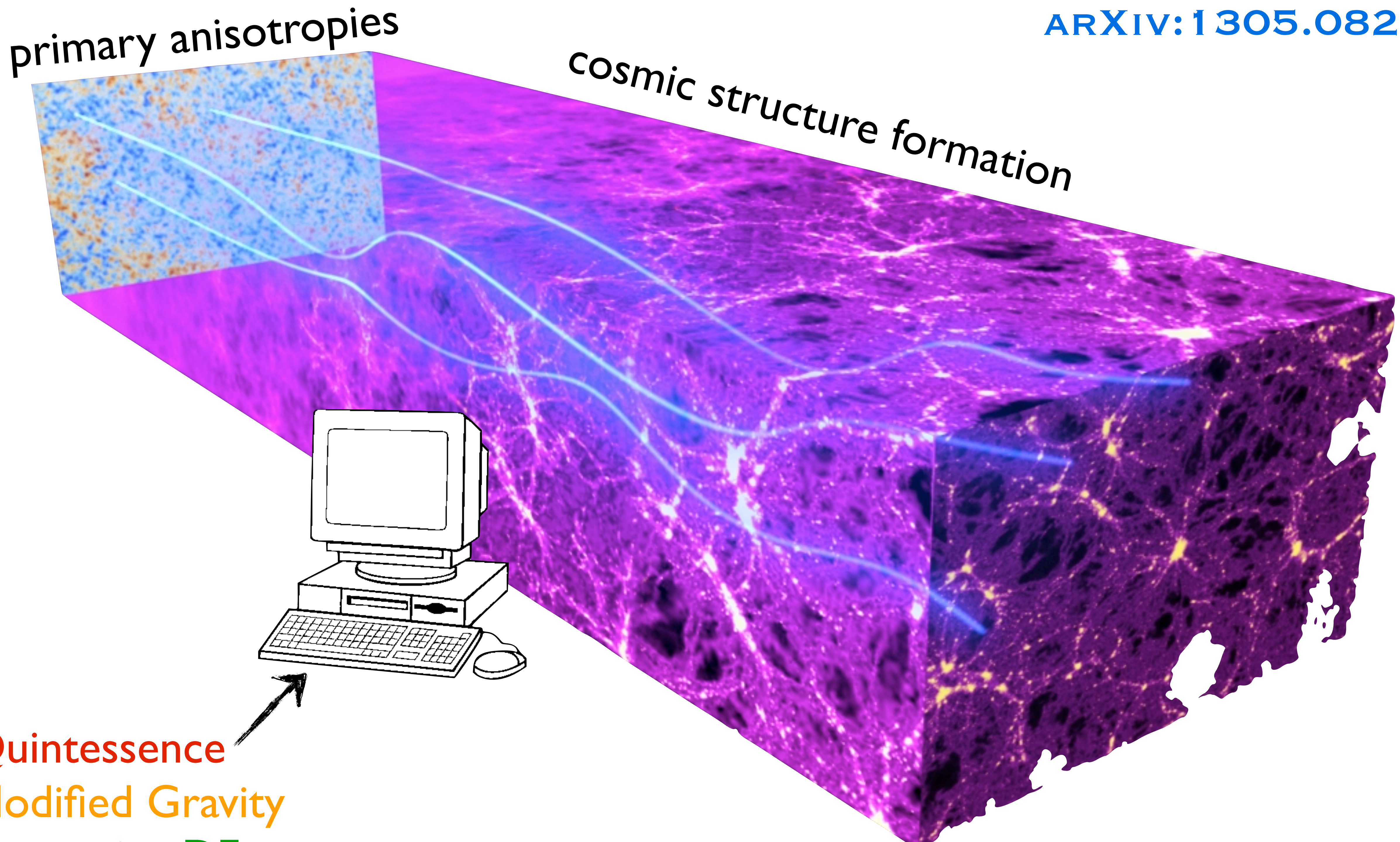

Quintessence
Modified Gravity
Interacting DE
Cold DE

Interacting Dark Energy

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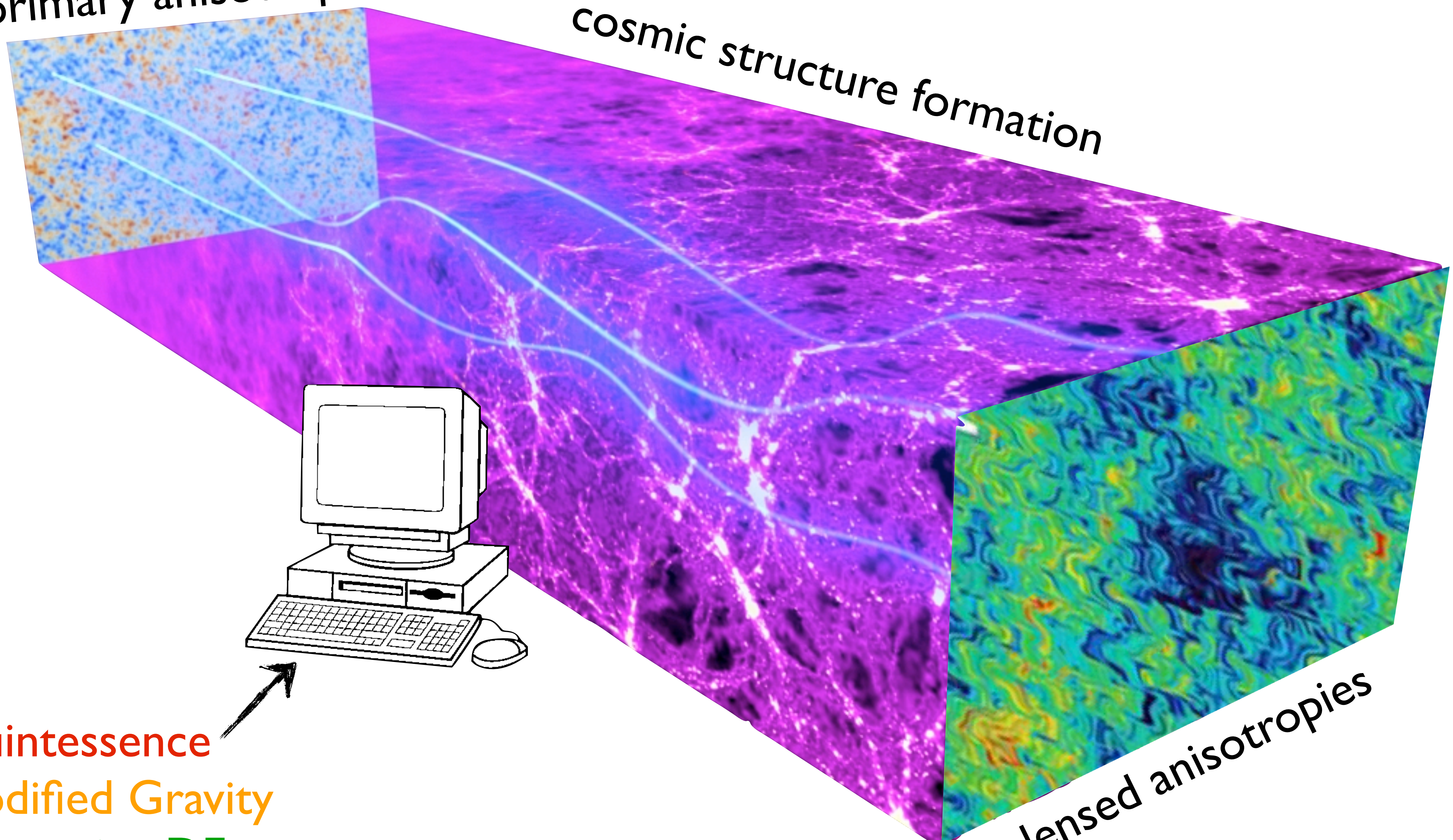

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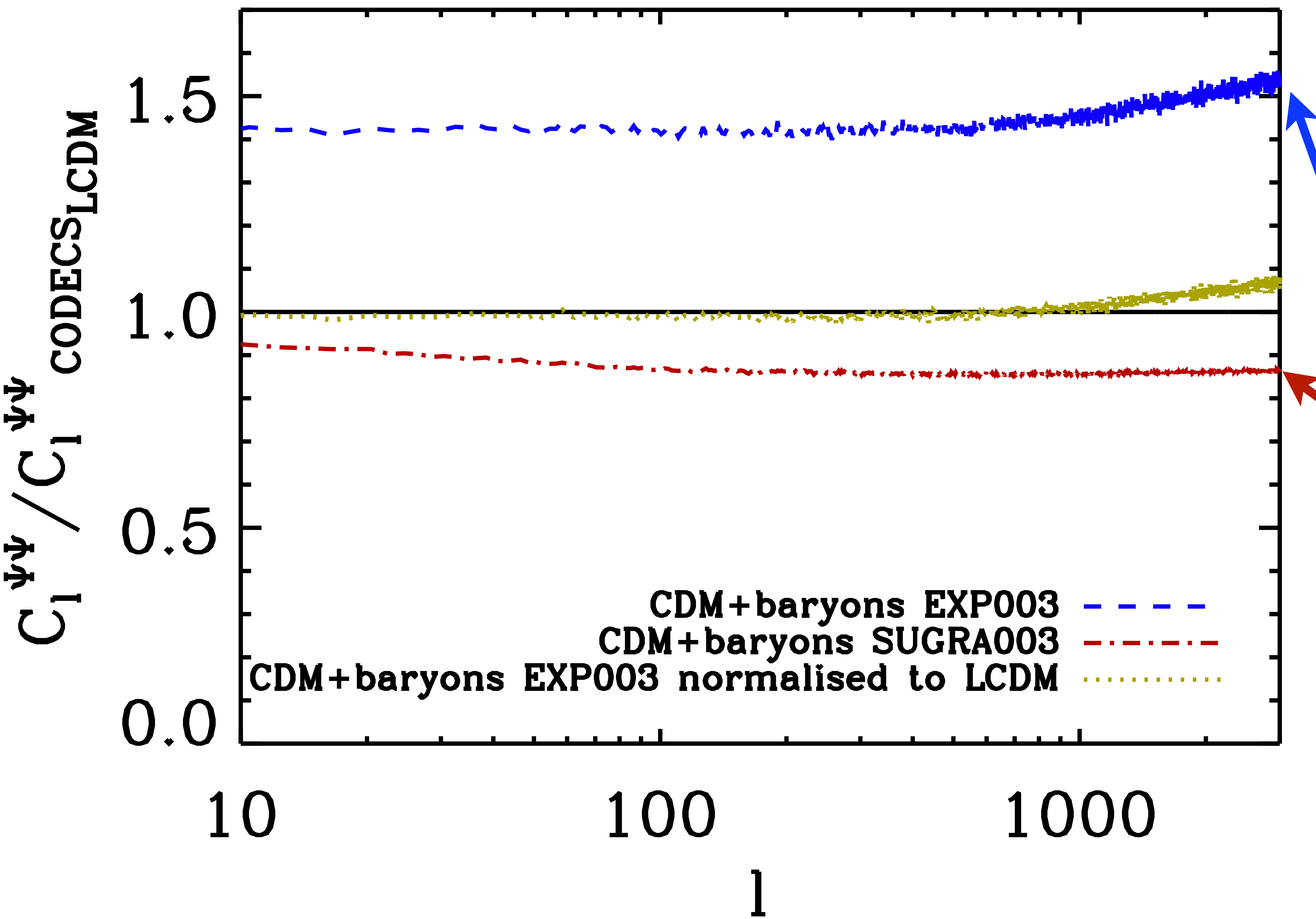

Different cDE models produce a different lensing signal on the CMB

CARBONE ET AL., 2013, ARXIV:1305.0829

Interacting Dark Energy

Non-universal couplings: Growing Neutrinos (Amendola, MB, Wetterich 2007)

The type of fifth-force is the same as for coupled quintessence, but involves massive neutrinos, and requires a much larger coupling such that the scalar force results orders of magnitude larger than gravity

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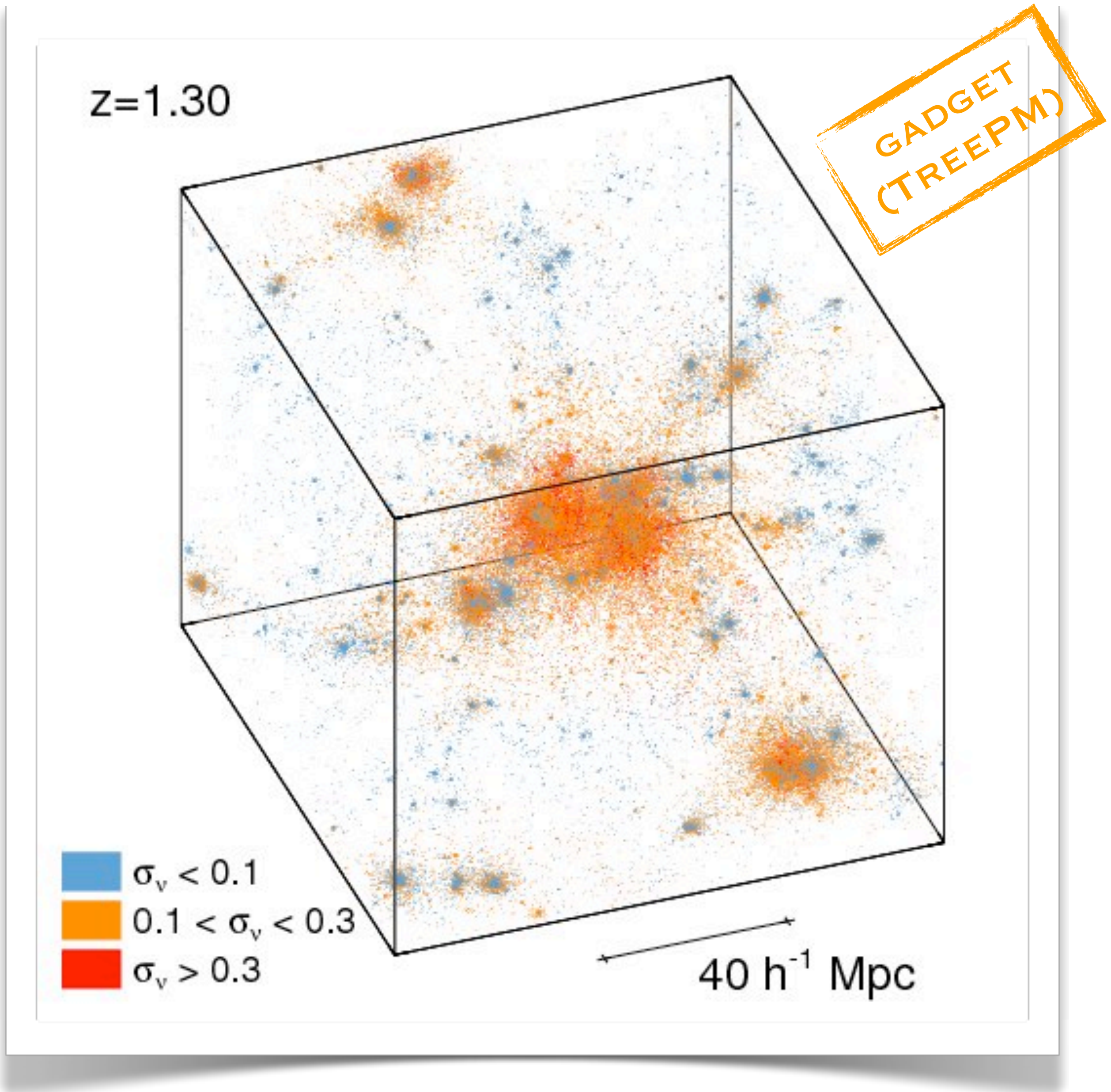

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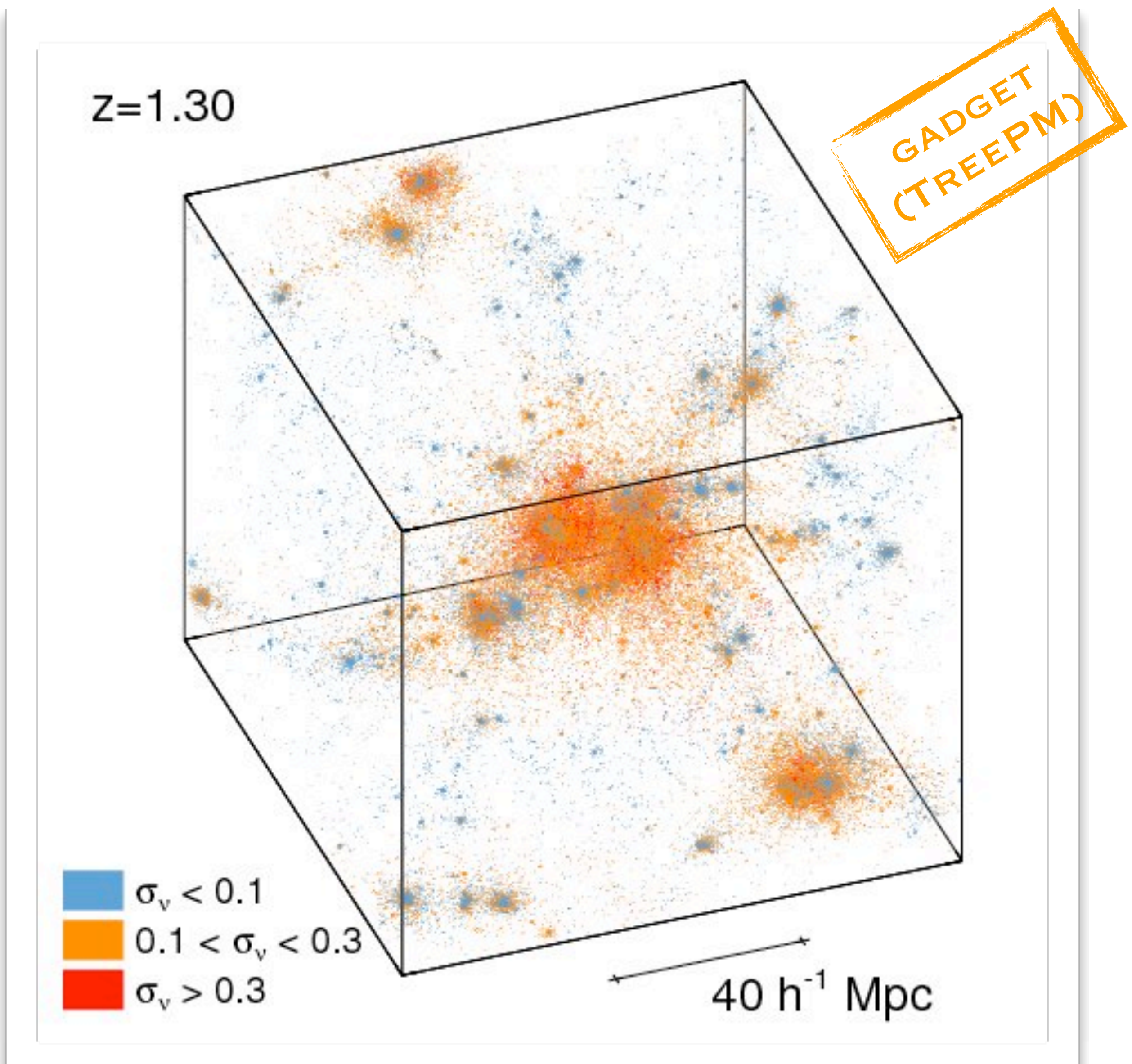

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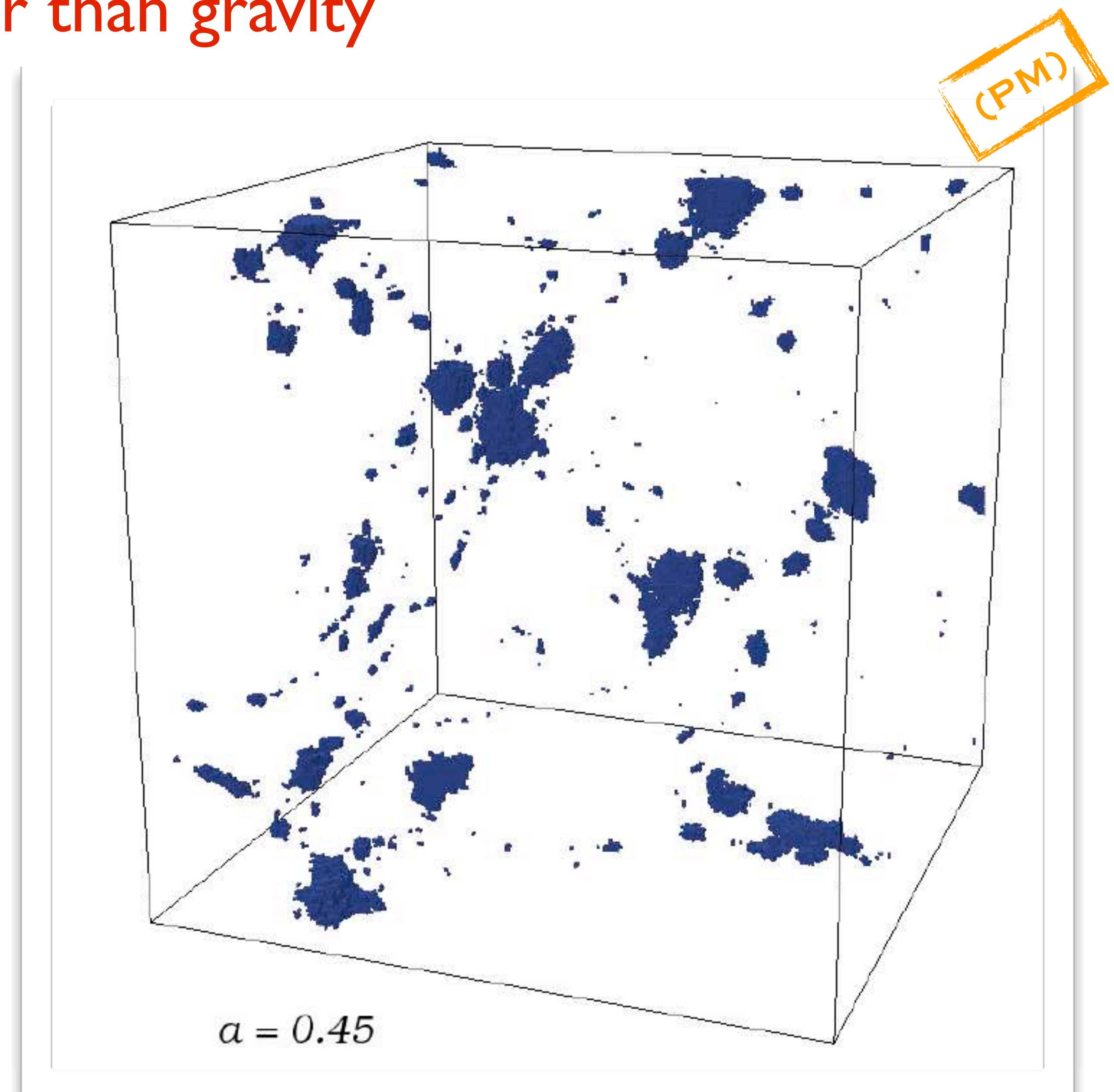

Ayaita, Weber, Wetterich 2011

NON-LINEAR STRUCTURE FORMATION IN MODIFIED GRAVITY MODELS

Modified Gravity

Universal couplings: Extended Quintessence, f(R), Symmetron, Dilaton, et al.

$$\nabla^2 \delta\phi = F(\delta\phi) + \beta(\phi)\delta\rho_M$$

where F is a nonlinear function: a nonlinear Poisson equation to solve!!

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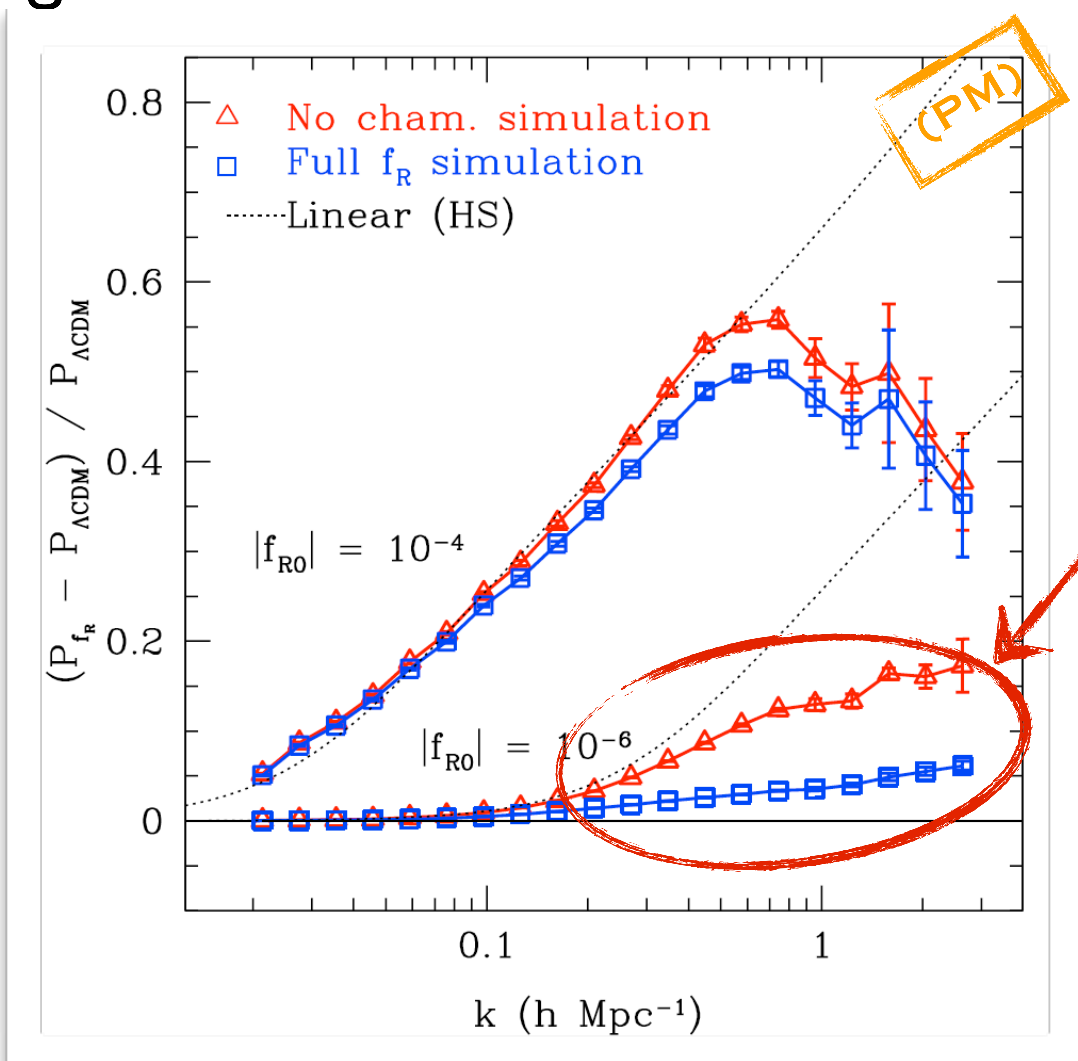
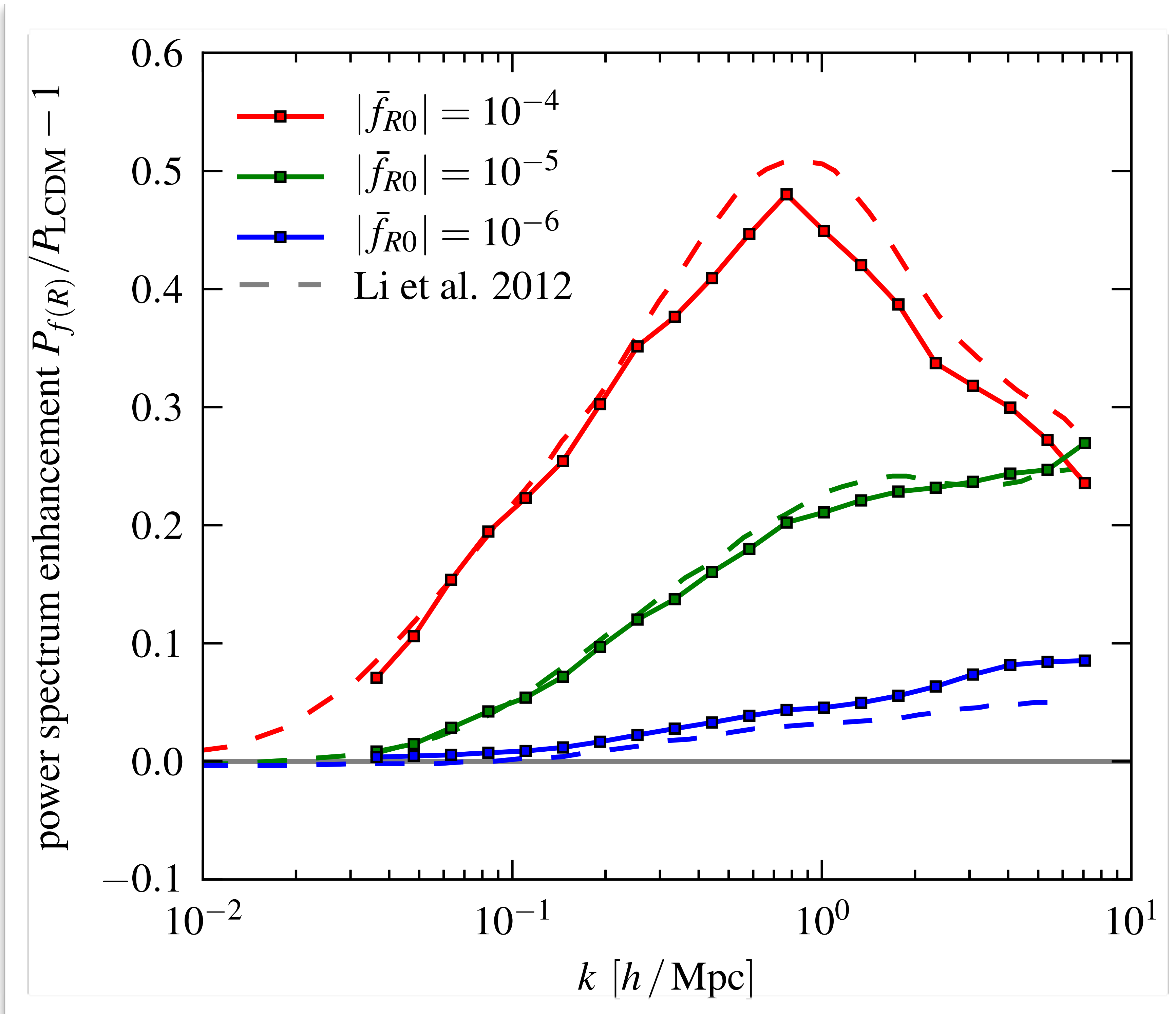
First simulations by Oyaizu 2008; Oyaizu, Lima, Hu 2008; Schmidt et al 2009 using an iterative scheme within a fix-grid PM code



The scalar fifth-force is suppressed in high-density regions according to the solution of the nonlinear Poisson equation for δϕ. The screening mechanism (in this case a Chameleon effect) is more efficient for lower values of $|f_{R0}|$

Modified Gravity

MG-GADGET



PUCHWEIN, MB, SPRINGEL, 2013

Modified Gravity

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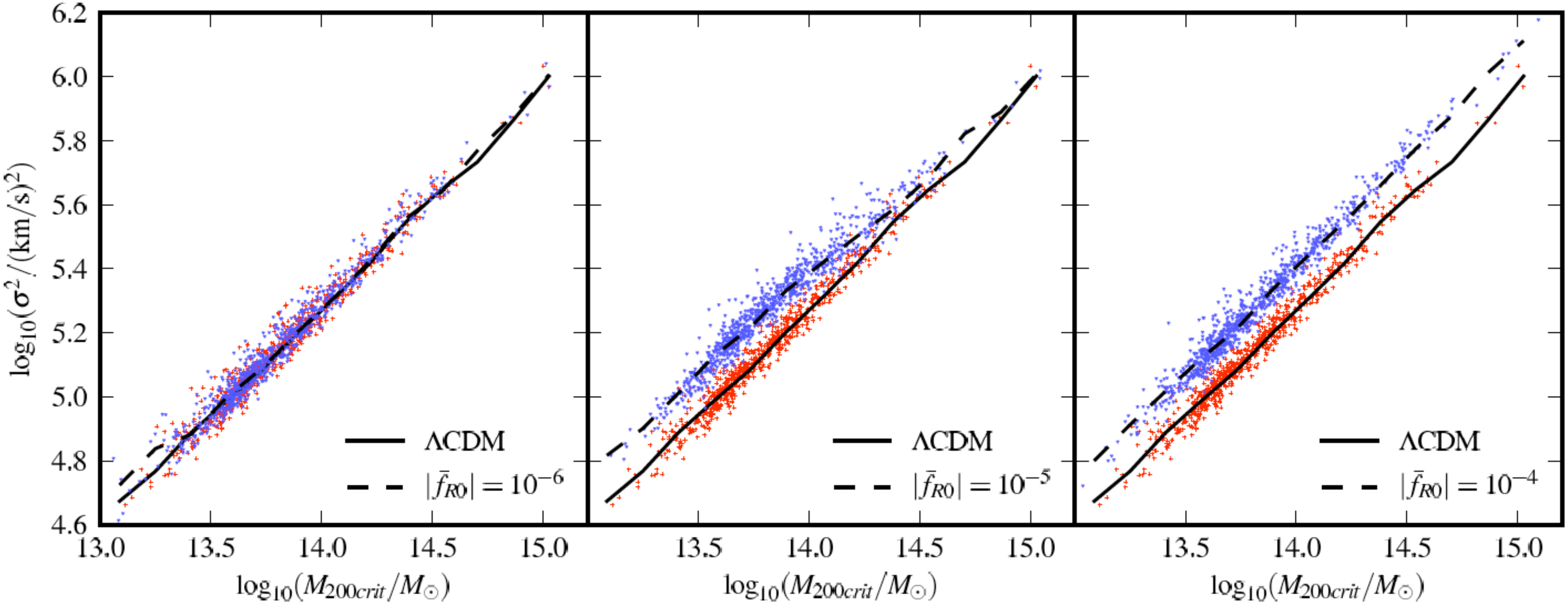

Dynamical vs. true masses of groups and clusters

ARNOLD ET AL. 2014

Recap Lecture 3

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Dark Energy can affect structure formation, through the background:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_M a^{-3} + (1 - \Omega_M)\exp\left(-3\int_1^a \frac{1 + w(a')}{a'}da'\right)$$

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$$\nabla^2 \Phi_g = -4\pi G(\delta\rho_M + \delta\rho_{\mathrm{DE}})$$

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or in the case DE is thought as a modification of GR, as in *f(R)*

$$\ddot{\delta}_{\mathrm{M}} + 2H\dot{\delta}_{\mathrm{M}} - \frac{4}{3}\frac{3}{2}\tilde{\Omega}_{\mathrm{M}}\delta_{\mathrm{M}} \simeq 0$$

Recap Lecture 3

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Cosmological N-body simulations allow to make predictions of non-linear structure formation also for non-standard models of Dark Energy or for Modified Gravity theories

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Cosmological N-body simulations allow to make predictions of non-linear structure formation also for non-standard models of Dark Energy or for Modified Gravity theories

Cosmological simulations also allow to break observational degeneracies of the DE features with other cosmological or astrophysical parameters